Prediction of the penetrometer resistance of soils with models with few parameters

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Abstract

The objective of this paper is to determine to what extent pedotransfer functions, PTFs, can be developed that have few coefficients and which are insensitive to soil type. The use of non-linear PTFs to predict penetrometer resistance of soils from their water status (matric potential, \(\psi\) and degree of saturation, \(S\)) and bulk density, \(\rho\), appears to require that some other soil property, such as sand content, is known. The use of a logarithmic transformation on the dependent variable, \(Q\) and the independent variables, either \(\psi\) or \(S\), \(\psi\) has two effects. Firstly, it linearizes the data and secondly it removes the increasing trend in the residuals of \(Q\). A pedotransfer function derived from fitting \(\log_{10} Q\) to \(\log_{10} S\psi\) and \(\rho\) had 3 parameters that were insensitive to soil type. However, to predict \(Q\) on its natural scale, back-transformed values require correction for bias.

There is evidence that \(S\psi\) is a better descriptor of soil water status than \(\psi\) alone with respect to predicting penetrometer resistance. We show that the use of \(S\psi\) is preferable for both statistical and physically based reasons. However, we also show that matric potential can work well when using PTFs to predict the strength of soil in the field given the variability in field measurements. We demonstrate how a PTF can be used to predict values of the strength of field soil measured independently.

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Keywords: Penetrometer resistance; Matric potential; Effective stress; Bulk density; Pedotransfer functions

1. Introduction

1.1. Penetrometer resistance and root growth pressure

The early use of penetrometers was frequently associated with the problem of trafficability, but recently penetrometer resistance has become an important measurement for assessing whether roots can penetrate soil (Bengough and Mullins, 1990; Whalley et al., 2005a; To and Kay, 2005). It is now understood that the resistance of a soil to a penetrometer can be a factor of up to 3 times greater than the force which a root needs to exert to penetrate the same soil (Bengough and Mullins, 1990). The greater resistance of soil to a penetrometer in comparison with a root can be explained largely by less frictional resistance between roots and soil in comparison with metal and soil (Bengough and McKenzie, 1997). Although lubricated penetrometers have been shown to have lower penetrometer resistance (Tollner and Verma, 1987), a more practicable solution to take account of the soil to metal friction when measuring penetrometer resistance is to rotate the penetrometer (Bengough et al., 1997). Typically the fixed penetrometer has a resistance to penetration two to three times higher than that of a rotating penetrometer (Whalley et al., 2005a). This is close to the ratio between the resistance to a fixed penetrometer and the pressure a root exerts to penetrate the same soil (Bengough and Mullins, 1991). Analysis of the relationship between penetrometer resistance and root growth pressure has increased confidence in the use of penetrometers to assess the rooting environment in soil. It is now generally accepted that a fixed penetrometer resistance in excess of 2.5 MPa will seriously restrict root elongation (e.g. Groenevelt et al., 2001).

1.2. A theoretical account of penetrometer resistance

To our knowledge the explanation of Farrell and Greacen (1966) remains the clearest account of a theory describing the relationship between soil properties and penetrometer resistance. The different zones of failure that Farrell and Greacen proposed...
around a penetrometer are shown in Fig. 1A. In the first of these zones (i) soil is compressed to its minimum voids ratio (or maximum bulk density), in the second zone (ii) the soil fails plastically and in the outer zone (iii) the soil behaves as an elastic material. Farrell and Greacen (1966) obtained an expression for $R$ in Fig. 1A by equating the volume of voids before and after the passage of the penetrometer. By solving the equations numerically, they obtained the radial pressure needed to expand a cavity in soil to accommodate the penetrometer. Although the model of Farrell and Greacen (1966) was effective at describing the penetrometer resistance (Fig. 1B), it is not useful in a practical sense because it requires data from the compression characteristic (Gregory et al., 2006) and the Mohr failure envelope which can only be found from comprehensive testing of the soil.

1.3. Experimental observations of how soil properties affect penetrometer resistance

Many accounts of the early use of penetrometers were concerned with inferring soil properties from penetrometer measurements of soil strength (e.g. Mulqueen et al., 1977) and to some extent this interest continues today (e.g. Hernanz et al., 2000). This work is instructive because it provides a detailed account of the relationship between soil properties such as shear, compressive and tensile strength and penetrometer resistance. Of particular value is an analysis of how these soil factors, which together determine penetrometer resistance, vary with soil type, water content and bulk density. Mulqueen et al. (1977) suggested that the mode of soil failure due to a penetrometer is a function of soil water content. At high water contents, they suggested that the soil failed plastically and the penetrometer resistance was insensitive to bulk density. At intermediate water contents, soil failed by a combination of compression and shear, and at low water contents, failure was related to the internal friction of soil and was sensitive to bulk density. They concluded that because of this complexity, it was unlikely that penetrometer measurements could be used to infer specific soil strength parameters. However, this does not affect the main current use of penetrometer data to estimate the ease

![Fig. 1. A. Soil failure zones ahead of a penetrometer being pushed into soil reproduced from Farrell and Greacen (1966). B. A comparison of modelled and measured penetrometer pressure. The modelled penetrometer pressure was that obtained by Farrell and Greacen (1966) with a mechanistic model of a soil failure ahead of a penetrometer.](image-url)
with which roots can penetrate soil (see Section 1.1). The work of Mulqueen et al. (1977) and others is useful because it can help us to develop simple relationships between easily measured, logged or predicted soil properties (e.g. water content and matric potential) and penetrometer resistance, and hence the resistance to root penetration, which is more difficult to measure and cannot be logged.

1.4. Predicting penetrometer resistance with pedotransfer functions, PTFs

A key property of a useful PTF is that it must be applicable to a wide range of soils and it must predict properties of soils that were not included in the development data set. Recently To and Kay (2005) described how a PTF to predict penetrometer resistance, \( Q \), could be derived from soil properties that change with time (e.g. matric potential, water content and bulk density) as well as those soil properties that are relatively constant with time such as texture and organic matter content. The PTF that fitted their data the best is written as

\[
Q = a\psi^b - c\psi
\]  

where \( \psi \) is the matric potential and \( a, b, \) and \( c \) were functions of texture, organic matter content and bulk density. For relatively wet soils with low density

\[
\log_{10} Q = a\log_{10} \sigma + b
\]

where (for low density soils) effective stress, \( \sigma \), can be approximated by

\[
\sigma = S\psi
\]

in which \( S \) is the degree of saturation obtained by dividing soil water content, \( \theta \), with the water content of the saturated soil (see Whalley et al., 2005a). In Eq. (2), \( a \) and \( b \) were not sensitive to soil type provided the bulk density of the soil was low.

The purpose of this paper is to use published data from laboratory and field experiments to explore the extent to which PTFs for soil penetrometer resistance can be developed that have the minimum number of parameters and that do not depend on soil type. We also explore the success of different curve fitting approaches at predicting penetrometer resistance from knowledge of soil water status and soil density.

2. Materials and methods

2.1. The laboratory data

To and Kay (2005) have collected a large set of data for undisturbed soils and it is used here for curve fitting and model testing. Briefly, the particle size classes, organic carbon content and density of these soils are given by To and Kay (2005), but reproduced here for ease of reference (Table 1). These soils were collected in cores at depths of 5, 15 and 25 cm from 5 sites in western Ontario in 1997 and 5 and 20 cm from seven sites in 1998. To and Kay (2005) report that all the sites were variable in landscape and that they were planted with corn according to tillage practices practised on the farms. Some of the sites were in no-till for various lengths of time and others were in conventional tillage consisting of autumn ploughing followed by spring cultivation to make a seedbed.

Fig. 2. The penetrometer resistance of three soils (triangles: Parafiel loam; squares: Urbrane loam and circles: Coleraine clay) packed to bulk densities from 1.0 to 1.7 cm\(^{-3}\) plotted against degree of saturation (A), matric potential (B) and effective stress (C). These different measures of soil water status explained 0, 54% (\( P<0.001 \)) and 70% (\( P<0.001 \)) of the variance in penetrometer resistance respectively. This is a reanalysis of the data of Farrell and Greacen (1966) in Whalley et al. (2005).
A cone penetrometer with a 30° cone angle and a basal diameter of 4 mm was used to make penetrometer resistance measurements. The rate of penetration was 2 mm/min. The penetrometer resistance of the undisturbed cores was measured following equilibration at matric potentials of −1, −3, −6, −10, −33, −100 and −1500 kPa. In total, penetration resistance was determined on 717 cores. After the penetrometer resistance of each core was measured and the soil water content was determined. Bulk density, texture and organic carbon contents are shown in Table 1.

We chose to use the data from undisturbed soils to avoid artefacts that can result when cores are packed to a high density and to ensure as far as is possible that the combinations of density, texture and organic carbon contents reflect those that are likely to occur in a managed agricultural system.

We also used penetrometer resistance data from undisturbed cores at Roseworthy College, South Australia, reported by Groenevelt et al. (2001). This was a non-swelling soil and the resistance to penetration was measured with a 2 mm diameter cone with a penetration rate of 2 mm min\(^{-1}\) on soil equilibrated at −1, −3, −5, −10, −33, −55, −100, −500 and −1500 kPa. Similar data presented by Farrell and Greacen (1966) are also used in this paper. They measured the penetrometer resistance of three soils repacked to bulk densities between 1.0 and 1.7 g cm\(^{-3}\) which were equilibrated at matric potentials of −30 at −70 kPa.

### 2.2. Data from field experiments

Whalley et al. (2006a) measured the penetrometer resistance of loamy sand during the spring and early summer of 2004 in a field experiment to investigate the effects of soil drying on soil strength. Winter wheat was grown and the soil was either well-watered to keep it at a matric potential of −5 kPa at a depth of 20 cm, prevented from drying to less than −80 kPa, or it was not irrigated. The matric potentials were monitored at a depth of 20 cm with a combination of water-filled tensiometers and porous matrix sensors (Whalley et al., in press). The soil water content was also monitored at a depth of 20 cm with a dielectric soil moisture meter (Whalley et al., 2004).

### 3. Results and discussion

#### 3.1. Appropriate measures of soil water status to predict penetrometer resistance

Estimates of soil water status and soil density are both needed to predict \(Q\) in soil. Soil water status can be defined by \(\theta, S, \psi,\) or the product \(S\psi\). Effective stress can be used to predict the tensile strength of soils with relationships that can be applied successfully to a wide range of soils provided \(S\) is greater than 0.5 (Mullins, 2000). The deformation of soil by a penetrometer is more complex than simple tensile failure, but nevertheless, Whalley et al. (2005a) have shown that effective stress can be used in a single expression (see Eq. (2)) to provide a common prediction of the penetrometer resistance of soils with a low bulk density. The most striking benefit of the use of effective stress is to be seen from re-examination of the data of Farrell and Greacen (1966), reproduced in Fig. 2. The use of effective stress gave a better prediction of penetrometer resistance than matric potential, but the degree of saturation was not correlated with penetrometer resistance. For the soils studied by Farrell and Greacen (1966) the use of effective stress gave a good prediction of penetrometer resistance over a range of soil bulk densities, but this is an exception. The original use of effective stress, \(\sigma\), for soil in equilibrium with atmospheric pressure is described by Bishop’s equation (Bishop and Blight, 1963)

\[
\sigma = P - \gamma \psi
\]

where \(P\) is the total stress on a soil at a matric potential, \(\psi\) and at a degree of saturation \(S\). Here \(\chi\) is a factor that takes into account the number of water-filled menisci at \(\psi\). For tensile failure when \(S > 0.5\), \(\chi\) can be approximated by \(\chi = S\) (Mullins, 2000; Whalley et al., 2005a). In very loose soils, strength is determined almost entirely by soil water status; \(P\) is small so the strength of the soil is determined by the product \(\psi S\). Under these conditions effective stress, \(\sigma \approx \psi S\). This explains why the penetrometer pressure of low density soil can be predicted from \(\psi S\) by a single equation (e.g. Eq. (2)) with the same fitted coefficients for a wide range of soils. In dense soils, when total stress, \(P\), is high the effective stress, \(\sigma\), which characterizes the intrinsic strength of soil, depends on both \(\psi\) which is the component of strength due to soil water status as

### Table 2a

The linear and non-linear models fitted to penetrometer resistance, \(Q\) (kPa), where \(\sigma_w\) (kPa) is effective stress, % sand is the percentage of sand in the samples, \(\rho\) (g cm\(^{-3}\)) is the bulk density, \(\psi\) (kPa) is the moisture potential and \(a, b, c\) and \(d\) are constants

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>(\log_{10}(\sigma) = \alpha \log_{10}(\sigma_w) + \beta + c)</td>
<td>(8)</td>
</tr>
<tr>
<td>(\log_{10}(\sigma) = \alpha \log_{10}(\psi) + \beta + c(\text{OC}) + d)</td>
<td>(9)</td>
</tr>
<tr>
<td>(\log_{10}(\sigma) = \alpha \log_{10}(\sigma_w) + \beta + c\times%\text{sand} + d)</td>
<td>(10)</td>
</tr>
<tr>
<td>(Q = \sigma^{\alpha^{OC}})</td>
<td>(11)</td>
</tr>
<tr>
<td>(Q = \sigma^{\alpha^{OC}\times%\text{sand} + d})</td>
<td>(12)</td>
</tr>
<tr>
<td>(Q = \psi^{\beta^{OC}})</td>
<td>(13)</td>
</tr>
<tr>
<td>(Q = \psi^{\beta^{OC}\times%\text{sand} + d})</td>
<td>(14)</td>
</tr>
</tbody>
</table>
well as $P$. For clarity we will define $\sigma_w = \psi S$ to identify it as the component of soil strength (or total stress) that is due to the soil water status.

The use of soil water content to predict penetrometer resistance (e.g. Grumwald et al., 2001) is likely to result in functions that have a greater dependence on soil type than would be the case if either $\psi$ or $\sigma_w$ were to be used. However, in many cases soil water content may be the only available measure of soil water status.

### 3.3. Penetrometer resistance as a function of soil water status and soil density

For low density soils the function obtained by Whalley et al. (2005a)

$$\log_{10} Q = 0.76\log_{10} \sigma + \log_{10} 62$$

(7)

gives good agreement with data from the To and Kay (2005) data set for soils with a bulk density less than 1.2 g/cm$^3$ (Fig. 4) and also those data reported by Groenevelt et al. (2001) and Farrell and Greacen (1966). Here we note that $\sigma \approx \sigma_w$. However, these bulk densities are unrealistically low for most situations.

With the data of To and Kay (2005), for undisturbed soil, the simple model of Whalley et al. (2005a) has been extended to include soil bulk density and the new model can be written as

$$\log_{10} Q = 0.35(\pm 0.009)\log_{10} \sigma_w + 0.93(\pm 0.0572)\rho + 1.2623(\pm 0.0832)$$

(8)

This explains 74.8% of the variance in $\log_{10} Q$ (Fig. 5) and the standard errors of the fitted parameters are given in brackets.

![Fig. 4. Prediction of $\log_{10} Q$ from effective stress using the relationship for low density soils developed by Whalley et al. (2005a) (solid line). Data plotted are those published by To and Kay (2005) for a bulk densities less than 1.2 g cm$^{-3}$ and matric potentials of −1.5 MPa or less (●). The data published by Groenevelt et al. (2001) (○) and Farrell and Greacen (1966) (□) are also plotted.](image-url)
If log$_{10}$ ψ is used to explain the variation in log$_{10}$ Q, both soil density and organic carbon content (Eq. (9) in Table 2a) were needed in the regression to explain 74.3% of the variance in log$_{10}$ Q. These extra variables decrease the number of degrees of freedom available to test the residual mean square and decrease the significance of the relationship (Table 2b).

The relationships between Q and soil water status and density are non-linear. However, they can be linearized by log transformation (Eqs. (8)–(10) in Table 2a). Log transformation before fitting is statistically desirable because it eliminates the increase in the variance of Q with increasing Q. However, the prediction of log$_{10}$ Q rather than Q may be less appealing. In Table 2a we have listed two linear models and their non-linear counterparts to relate Q to σ$_w$, ρ, and in one case also the sand content of the soil. A non-linear model to predict Q from matric potential and soil bulk density is also listed. Including the sand content of the samples gives a significant improvement in all cases but the benefit is much greater in the two non-linear models (12) and (14). There is a benefit in using σ$_w$ instead of, ψ since the smaller RMS with models (11) and (12) indicates that it is better to use these than the corresponding models (13) and (14). When limited data on soil properties is available, we recommend that model (8) in Table 2a is used to predict log$_{10}$ Q because it has parameters that are not sensitive to soil type (i.e. it does not contain OC or sand content), and its inputs are either easily measured with sensors (i.e. S and ψ) or they depend on properties such as bulk density that change slowly in rigid soils or that can be calculated from shrinkage in soils with a high clay content. When more data on the soil is available such as texture and organic carbon content then the functions such as model (10 or 12) or those described by To and Kay (2005) can be used to estimate Q.

Transforming to logarithm enabled us to fit a model to data where the variance in the independent variable increases with the dependent variable(s), here stress σ$_w$ or ψ and ρ. The residuals derived in this way have no trend whereas the residuals on the natural scale increase with σ$_w$ or ψ. Although the parameters in Eq. (8) are therefore unbiased, the relationship is not helpful where there is a need to predict Q rather than log$_{10}$ Q. In transforming back, bias is introduced and in general the predicted values will be too low. The simplest way to avoid this is to multiply the back transformed value (i.e. taking the anti-logarithm log$_{10}$ Q predicted in Eq. (8)), by the mean of the anti-logarithm of the residuals,

$$\frac{1}{n} \sum \log_{10}(\hat{Q} - \hat{Q})$$

where $\hat{Q}$ is the fitted value. Using the data of To and Kay (2005) this factor has a value of 1.097. Strictly, multiplication of Eq. (8) with Eq. (15) is valid only where there is no trend in the residuals, which is not the case here. A better correction can be applied by taking account of how the residuals vary with σ$_w$ and ρ but this may not work well where there is little or no replication or if the relationship is intended for extrapolation outside the range used to derive it. Expression (15) is much more robust under these circumstances even though some bias will remain. Essentially Eq. (15) is an empirically-derived correction factor that is widely applicable to soils similar to the ones studied here. A PTF based on its use will be improved by the collection of more data as will Eq. (8). The combined use of Eqs. (8) and (15) to predict Q provides a PTF with only 3 parameters all of which are insensitive to soil type.

### 3.4. Predicting the strength of soil in the field

The discussion to this point has been concerned with the use of PTFs to predict the strength of soils in cores. However, for these to be useful they must also give a prediction of the penetrometer resistance in the field. The field environment is more complicated than that of soil cores because, even for a soil profile with a uniform water content and density at all depths, penetrometer resistance will tend to increase with depth because of the difficulty in moving soil particles that support the weight of the soil above (Fig. 6). However, the strength of the surface
layer soil is almost entirely due to its density and water status. The penetrometer resistance of soil measured at a depth of 20 cm in a field experiment (see Whalley et al., 2006a) has been predicted with Eq. (8) which was derived from the independent data of To and Kay (2005) and also with their PTF (Eq. (1)). The combined use of Eqs. (8) and (15) has the advantage that prediction of $Q$ can be made with a model that has only 3 parameters none of which is dependent on soil type. The prediction of $Q$ with this approach differs by $-0.039$ MPa on average from the measured values with an SE of 0.039 and this error is not significantly different from zero. To and Kay’s (2005) model differs by 0.22 MPa on average with an SE of 0.045 but this small difference does differ significantly from zero (Fig. 7).

It should be noted that the measured penetrometer data shown in Fig. 7 was obtained from soils with values of $S$ greater than 0.5 with the exception of two cases when $S$ was close to 0.45. If $S$ becomes very small it is likely that the To and Kay (2005) PTF would give a better prediction of $Q$. However, these very dry soils are likely to be too strong for any root elongation.

3.5. Soil-type dependence in PTFs

The PTF developed by To and Kay (2005) (see Eq. (1)) has parameters that depend on soil texture, organic matter content and bulk density. Predictions of $Q$ can then be made for various soils as a function of $\psi$ for different soils using the appropriate set of parameters. In Eq. (8), $Q$ is simply a function of $\sigma_w$ and $\rho$. Although we have shown that this simpler PTF can work well for an independent data set obtained with sandy soil (Fig. 7), the extent to which it can account for soil-type dependence in the relationship between $Q$ and $\psi$ merits discussion. In Eq. (8) for a given value of $\psi$, there will be a spread in values of $\sigma_w$ because $S$ will be different for different soils according to the water release characteristic. In Fig. 8, the fitted values of $\log_{10} Q$ and the back transformed estimates of $Q$ are plotted against representative measured data for clay and sandy soils. This shows that the variation in $Q$ with soil type can be accounted for in PTFs that are based simply on the use of $\sigma_w$ and bulk density.

3.6. Uncertainty analysis

Eq. (8) is a relatively simple function, so that amplification of input errors is unlikely in its use. Nevertheless we examined the effects of combinations of likely values of the inputs (bulk density, $\rho$ 1.2–1.65; $\sigma_w$ 25–200 kPa) on the predicted value of $\log_{10} Q$ using Eq. (8). These results are summarised in Table 3. The resultant variance in $\log_{10} Q$ is close to the input variance in bulk density and less than the variance in $\sigma_w$ emphasising the importance of precise estimates of the former. Eq. (8) does not appear to amplify variation during the propagation of input error into output. However, $\rho$ and $\sigma_w$ may be weakly correlated in data of To and Kay (2005) and also with their PTF (Eq. (1)).

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Output</th>
</tr>
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<tbody>
<tr>
<td>$\rho$</td>
<td>$\log_{10} \sigma_w$</td>
</tr>
<tr>
<td>Mean</td>
<td>1.425</td>
</tr>
<tr>
<td>Median</td>
<td>1.425</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0209</td>
</tr>
</tbody>
</table>

Table 3
The results of a Monte-Carlo randomisation of the inputs ($\rho$ and $\sigma_w$) to investigate the propagation of errors in Eq. (8) (uncertainty analysis)
practice and if so this might alter the output variance slightly. The data in Table 3 were obtained by full Monte-Carlo randomisation of the inputs.

4. Conclusions

If little is known about the soil, then \( \log_{10} Q \) can be predicted with Eq. (8), provided both soil density and soil water status are known. We have shown how to correct for bias in estimates of \( Q \) obtained by back-transformation. However, an estimate of the mean of the anti-logarithm of the residual error is needed, although Eq. (15) is likely to be widely applicable. If soil texture and organic matter content are known, non-linear functions can be used to predict \( Q \) on its natural scale. Although we have shown that \( \sigma_w \) is preferable to the use of matric potential, because it either describes more of the variance or requires the use of fewer parameters, \( Q \) can be predicted from \( \psi \) if a reduction in precision is acceptable or if additional information on the texture and soil organic carbon content is available.

Acknowledgements

Rothamsted Research is grant-aided by the Biotechnology and Biological Sciences Research Council. We thank Dr. C.E. Mullins for helpful comments on an earlier draft of this paper.

References


