Stabilization of porous reservoir by stressed filter

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Received 15 January 2004; accepted 2 November 2005

Abstract

A mechanical stability theory for stressed wellbore filters was developed. The stressed-state solution was obtained for the stressed filter in a uniform porous reservoir by considering the nonlinearity of fluid flow and compressibility of a fluid. It was found that, in typical cases, a stressed filter stabilizes a poorly consolidated reservoir rock. A formula was derived for the critical production rate, at which the rock experiences failure. No destruction of the reservoir rock was observed when using a stressed filter with parameters similar to those of wire-wrapped screens under underground gas storage conditions. For low Young’s modulus values, the rock remains in an elastic condition; for high Young’s modulus conditions, a thin plastic zone may develop adjacent to the stressed filter, but without impairing the overall stability of the zone. Even in the case of a significant permeability reduction arising from particle movement or saturation changes, the stressed filter stabilizes the bed. During water and gas co-production, the stressed filter appears to sustain wall stability even at high levels of water saturation.

The theory that is presented is valid both for inflow and for outflow. The theory can be used for the design of stressed filters and completion of wellbores for both oil and gas fields, as well as in the case of underground gas storage facilities. © 2005 Elsevier B.V. All rights reserved.

Keywords: Wellbore filter; Sand control; Fluid flow; Porous reservoir; Stress; Production rate

1. Introduction

The physics of shear yield and potential collapse of reservoir rock around a perforation channel or uncased wellbore has been addressed by many scientist (Bratli and Risnes, 1981; Risnes et al., 1982; Morita, 1994; Van den Hock et al., 1996; Behrmann et al., 1997; and, for repeated hydraulic loading by Wang and Dusseault, 1994; for gas reservoirs by Weingarten and Perkins, 1995; for inclined wellbores by Wang and Dusseault, 1996; Wiprut and Zoback, 1998; for horizontal wellbores by Kooijman et al., 1996; for water breakthrough by Bruno et al., 1996; Skjaerstein et al., 1998; and, for nonlinearity of fluid flow by Pyatakhin et al., 2003; Pyatakhin and Kazaryan, 2004).

Research of sand production and sand control in poorly consolidated sandstones where filters (sand screens, slotted liners, perforated liners, etc., that have high fluid permeabilities) are used is of considerable interest. Many of the wells are completed with a gravel pack located in the annular space between the wellbore wall and the filter (Gavrilko and Alekseev, 1976). The new technologies include the use of expandable filters that allow one to minimize the gap between the liner and the borehole wall and to minimize cavities in gravel (Chatterji et al., 2003).

The objective of this work was to analyze the characteristics of stressed filter (SF). These filters operate in such a way that they create mechanical stresses against the reservoir and thus prevent collapse of the wellbore.
wall. The stressed filter can include gravel packing between the filter and a wellbore wall.

The stresses in the reservoir, near the wellbore and in the filter, were analyzed. The mechanisms for destruction of a bed were discussed. Similar problems are of the great theoretical, practical and economic importance because their solution will allow one to optimize the technology of well completions, the production conditions of well, and to reduce the frequency of problems during production.

Spatial dependencies of the principal stresses acting on the stressed filter and on the reservoir near the wellbore are found. The SF acts as a permeable skeleton, which applies a compressive radial stress to the reservoir rock near the wellbore and provides stability for unconsolidated sand. For underground gas storage (UGS), the direction of flow changes because of cyclic operation and the stressed filters maintain reservoir stability. Fluid flow does not influence the distribution of principal stresses in a vicinity of SF (even at large relative permeability to the hydrocarbon phase and cohesion of reservoir particles from higher water saturation. During high production rates, the decrease in hydraulic conductivity on the ratio of the open area of the filter to the surface area is given in Fig. 65 in Gavrilko and Alekseev (1976) book. This ratio of the wire-wrapped filters used on UGS wellbores is 9–18% depending on type of the filter (Kazaryan et al., 1998). From Fig. 65 at this ratio of 10%, \( k_f \approx 1.7 \text{ cm s}^{-1} \) is obtained; as a result the permeability of the filter is about \( 1.7 \times 10^8 \) Darcy. Calculations show that pressure loss across the filters can be neglected at such high permeabilities. Thus, inside the filter the pressure is constant \( (p(r) = p_1) \).

The distribution of pressure in a vicinity of a cylindrical filter (where fluid flow is nonlinear and \( p_2 - p_1 < 0.2 p_2 \)) and including fluid compressibility is:

\[
p(r) = p_2 + A \ln \frac{r}{r_2} + B \left( \frac{1}{r_2} - \frac{1}{r} \right),
\]

\[
A = \frac{\mu q p_{at} z T}{2 \pi k h p_2 T_0}; \quad B = \frac{\beta_D q |p_{at} - p_{at} T|}{4 \pi^2 h^2 p_2 T_0},
\]

where \( q \) is the wellbore production rate, \( p_{at} \), the atmospheric pressure, \( z \), the coefficient applied to ideal gas compressibility to account for nonlinear behavior at

2. Fluid flow near an SF

The dependence of permeability on pressure and stresses (Han and Dusseault, 2003) is neglected to obtain the distribution of pressure in a bed (Pyatakhin et al., 2003). The relationships obtained by Gavrilko and Alekseev (1976) are used to find the loss of pressure across a filter. Darcy’s law and a correlation of permeability \( (k) \) with hydraulic conductivity \( k_f \) (Bassiniev et al., 1993) were used:

\[
k = \frac{k_f \mu}{\rho g},
\]

where \( \mu \) is the fluid viscosity; \( \rho \), the density of fluid and \( g \), the acceleration of gravity, for an estimation of permeability of wellbore filters. The dependence of hydraulic conductivity on the ratio of the open area of the filter to the surface area is given in Fig. 65 in Gavrilko and Alekseev (1976) book. This ratio of the wire-wrapped filters used on UGS wellbores is 9–18% depending on type of the filter (Kazaryan et al., 1998). From Fig. 65 at this ratio of 10%, \( k_f \approx 1.7 \text{ cm s}^{-1} \) is obtained; as a result the permeability of the filter is about \( 1.7 \times 10^8 \) Darcy. Calculations show that pressure loss across the filters can be neglected at such high permeabilities. Thus, inside the filter the pressure is constant \( (p(r) = p_1) \).

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\]

where \( q \) is the wellbore production rate, \( p_{at} \), the atmospheric pressure, \( z \), the coefficient applied to ideal gas compressibility to account for nonlinear behavior at
high pressures, \( \beta_{D} \), the non-Darcy flow coefficient, \( \rho_{atm} \), the density of gas at atmospheric pressure, \( T_0 = 293 \text{ K} \) and \( T \), the reservoir temperature (Pyatakhin et al., 2003).

The results also are valid for oil wells with nonlinear fluid flow of an incompressible liquid such as oil, water, etc. The distribution of pressure in a reservoir is described by the same expression with different coefficients:

\[
\tilde{A} = \frac{\mu q}{2\pi kh} \quad \tilde{B} = \frac{\beta_{D} \rho q \bar{q}}{4\pi h^2}
\]  

\[ (4) \]

3. Elastic deformation of the stressed filter

The equation for radial displacement of the filter, \( s \), (assuming no other displacements) for fluid flow through the filter is (Risnes et al., 1982):

\[
(\lambda_t + 2G_t) \frac{d}{dr} \left( \frac{ds}{dr} + \frac{s}{r} \right) + \beta_t \frac{dp}{dr} = 0 \]  

\[ (5) \]

where \( \lambda_t \) and \( G_t \) are Lamé parameters of the filter; \( 1 - \beta_t \) is the ratio of a material and volume compressibilities of the filter, and for particular calculations used \( \beta_t = 1 \). This equation assumes plane strain, linear elastic material, and axisymmetric deformation. Solving Eq. (5) for displacement, \( s \), assuming constant pressure in the filter:

\[
s = \frac{c_1}{r} + c_2 r - \frac{\beta_t r p_1}{2(\lambda_t + 2G_t)}, \]  

\[ (6) \]

where \( c_1 \) and \( c_2 \) are constants of integration. Using expressions for components of elastic strain:

\[
\varepsilon^e = \frac{ds}{dr}
\]

\[
\varepsilon^g = \frac{s}{r}
\]  

\[ (7) \]

and assuming constant vertical strain \( \varepsilon^v = \varepsilon^v_0 \), the radial component of total stress is (Pyatakhin et al., 2003):

\[
\sigma_r = \lambda_t \varepsilon^e + 2G_t \varepsilon^e + \beta_t p = \tilde{\varepsilon}_1 - \frac{\varepsilon_2}{r^2} \]  

\[ (8) \]

Here \( \varepsilon^e = \varepsilon^v + \tilde{\varepsilon}_1 + \varepsilon^v_2 \), \( \varepsilon_1 \), and \( \varepsilon_2 \) are constants, which are determined from boundary conditions.

On the internal wall of the filter \( \sigma_r = p_1 \) at \( r = r_0 \) \( (r_0 = r_1 - \Delta) \), where \( \Delta \) is thickness of a wall of the filter. On the external wall of the filter \( \sigma_r = \sigma_{r0} \) at \( r = r_1 \). Using these conditions, the radial stress component in the filter is:

\[
\sigma_r(r) = \frac{\sigma_{r0} r_1^2 - p_1 r_0^2}{A(r_1 + r_0)} - \frac{(\sigma_{r0} - p_1) r_0^2 r_1^2}{A(r_1 + r_0) r^2} \]  

\[ (9) \]

To find the angular component of total stress, a balance condition of an element of the filter is used (Risnes et al., 1982):

\[
\sigma_\theta = \frac{r \, \frac{d\sigma_\theta}{dr} + \sigma_\theta}{A(r_1 + r_0)} - \frac{(\sigma_{r0} - p_1) r_0^2 r_1^2}{A(r_1 + r_0) r^2} \]  

\[ (10) \]

Thus, the stress distribution in the elastic filter material is obtained at the known radial stress on the external wall of the filter. These results are used to determine the boundary conditions for the stress-state solution in the bed adjoining the cylindrical SF.

To obtain the radial stress component, an expression for angular elastic strain of the filter on its external wall is required. From Hooke’s law, generalized for fluid flow (Risnes et al., 1982):

\[
\varepsilon^e_\theta = \frac{\sigma_{r0} - \nu_1 (\sigma_r + \sigma_z) - (1 - 2\nu_1) p}{E_1}, \]  

\[ (11) \]

where \( \nu_1 \) and \( E_1 \) are Poisson’s ratio and the Young’s modulus, respectively, of the filter (assuming that the effective vertical stress is equal to zero \( \sigma_z - p_1 = 0 \)):

\[
\varepsilon^e_\theta(r_1) = \frac{(\sigma_{r0} (r_1^2 + r_0^2) - 2p r_0^2}{(r_1 + r_0) A} - \nu_1 \sigma_{r0} - (1 - \nu_1) p_1} / E_1 \]  

\[ (12) \]

The following variants are possible for the stressed-state solution of the reservoir zone near the stressed filter:

1. Only elastic deformation occurs in the reservoir rock;
2. Adjacent to the stressed filter there is a region of plastic deformation with an outer elastoplastic boundary — \( r_c \).

4. Elastic deformation of a bed in a vicinity of stressed filter

The expressions for the stress components of elastic deformation in a bed near a cylindrical wellbore for nonlinear fluid flow were obtained by Pyatakhin et al. (2003). Those expressions (substituting \( r = r_1 \)) are used to find a boundary condition on the wall of the filter adjacent to the reservoir:

\[
\sigma_r = \sigma_{r0} \]  

\[ (13) \]

\[
\sigma_\theta = \frac{2\nu p_0 + (p_2 + p_1) (1 - 2\nu) - \sigma_{r0}}{1 - \nu} \]  

\[ (14) \]

where \( \nu \) is Poisson’s ratio of the sandstone and \( p_0 \), the overburden pressure.

At the border between the filter and the borehole wall, the total radial stress \( \sigma_{r0} \) and the displacement must be
continuous. As the displacement at the border is \( s = \varepsilon_0 r_1 \), the equality of angular elastic strains \( \varepsilon_0^a \) follows from equality of displacement at \( r = r_1 \). An expression for angular strain in the reservoir is obtained by substituting Eqs. (13) and (14) for the stresses at the border between the filter and the bed (at \( r = r_1 \) ) in Hooke’s law:

\[
\varepsilon_0^a(r_1) = \left[ p_0 v + p_2 (1 - 2v)(1 + v) \right] / E, \tag{15}
\]

where \( E \) is the Young’s modulus of the sandstone.

Equating the last expression for a bed (Eq. (15)) and the expression obtained above for angular strain of the filter (Eq. (12)), the boundary condition at elastic deformation of a reservoir is:

\[
\sigma_{r0} = \frac{a p_0 v + p_2 (1 - 2v) + p_1 \left( \frac{2r_0^2}{b} + 1 - v_f \right)}{1 - v} + \frac{r_1^2 + r_0^2}{b} - v_f \tag{16}
\]

where

\[
a = \frac{(1 + v)E_f}{E}; \quad b = A(r_1 + r_0). \tag{17}
\]

It is important to say that the SF applies compressive radial stress described by Eq. (16) to the rock. In practice this stress can, and should be, adjusted with respect to the requirements of a wellbore operation.

Compressive radial stress of the filter depends on its geometrical parameters such as radius, \( r_1 \), and wall thickness, \( A \), and also on the Poisson’s ratios and the relation of the Young’s modulus of the filter and rock, fluid bottom hole pressure and the overburden pressure. Formulas for elastic stress components in a bed taking into account a nonlinear fluid flow are:

\[
\sigma_t(r) = \frac{2vp_0 + \beta p_2 (1 - 2v) - (0.5 - v)\beta A \left( 1 - \frac{r_1^2}{r_2^2} \right)}{2(1 - v)} + \frac{r_1^2}{r_2^2} \sigma_{r0} + \frac{(1 - 2v)}{2(1 - v)} \beta \left[ p(r) - \frac{r_1^2}{r^2} p_1 + \left( \frac{r_1}{r - r_2} \right) B \right] \tag{18}
\]

\[
\sigma_0(r) = \frac{2vp_0 + \beta p_2 (1 - 2v) - (0.5 - v)\beta A \left( 1 + \frac{r_1^2}{r_2^2} \right)}{2(1 - v)} - \frac{r_1^2}{r_2^2} \sigma_{r0} - \frac{(1 - 2v)}{2(1 - v)} \beta \left[ p(r) + A + \frac{r_1^2}{r^2} p_1 + \left( \frac{r_1}{r - r_2} \right) B \right] \tag{19}
\]

\[
\sigma_z(r) = p_0 + \beta \frac{(1 - 2v)}{1 - v} \left[ p(r) - p_2 \right], \tag{20}
\]

where \( 1 - \beta \) is the ratio of a material and volume compressibilities of the sandstone, and for the calculations \( \beta = 1 \) yielded accurate results. Together with the formula for compressive radial stress of the filter (Eqs. (16) and (17)), they yield the elastic stresses in a vicinity of cylindrical SF.

Calculations of spatial dependencies of radial, angular and vertical stresses acting on a rock adjoining the filter (and of the pressure) were conducted using the theory that was developed. The initial data that were used for SF and UGS are listed in Table 1.

Numerical values of stresses were substituted into the Coulomb failure criterion, Fig. 2:

\[
\sigma_z - p = 2S_0 \tan \alpha + (\sigma_t - p) \tan^2 \alpha, \tag{21}
\]

where \( S_0 \) is the cohesive strength and \( \alpha \), the failure angle.

The analysis of the results show, that at a Young’s modulus of sandstone equal to \( A = 4 \times 10^4 \) MPa (Zheltov, 1966), \( S_0 = 0 \), and in absence of a fluid flow, the Coulomb failure criterion is disturbed near the wall of the stressed filter (Fig. 2). Thus, a rock close to the filter is in a plastic state.

The first new physical result is that fluid flow can lead to the disappearance of the plastic zone leaving only elastic deformation of sandstone in the reservoir (Figs. 2–4) near the stressed filter. For the perforations (Bratli and Risnes, 1981; Pyatakhin and Kazaryan, 2004), and for uncased wellbores (Risnes et al., 1982; Pyatakhin et al., 2003), the plastic region exists both with and without fluid flow, and an increase of the production rate results in an expansion of the plastic region.

The second physical result from use of a stressed filter is that the Coulomb failure criterion is not disturbed even at a low rock cohesion. Actually, from Fig. 2, the Coulomb failure criterion is not disturbed in a

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vicinity of the stressed filter at a production rate, $q$, equal to 2 millions m$^3$/day, even for unconsolidated sand with the cohesive strength $S_0 = 0$.

Consider the significant spread of literature data on sandstone mechanical properties. Nikolaevskiy (1996) reports Young’s modulus for a sandstone as $A = (0.2–7) \times 10^4$ MPa. Calculation of the stresses in the elastic zone for two values Young’s modulus ($A = 2 \times 10^4$ and $4 \times 10^4$ MPa) for the wire-wrapped SF (Zheltov, 1966; Kazaryan et al., 1998) is plotted in Fig. 3. In a reservoir with a low value of Young’s modulus (even at the absence of a fluid flow) only the elastic deformation region surrounds the stressed filter.

The value of radial and angular stress at some distance from the wellbore is determined by the Poisson’s ratio of the sandstone (Risnes et al., 1982; Pyatakhin et al., 2003). According to data in the literature, Poisson’s ratio for a sandstone varies from 0.4 to 0.13 (Nikolaevskiy, 1996; Zheltov, 1966). In Fig. 4, the stress calculations are for the vicinity of the wire-wrapped wellbore SF at a production rate of 1 million m$^3$/day and Poisson’s ratios equal to 0.35 and 0.13. At low Poisson’s ra-

Fig. 3. Stresses in porous material and fluid pressure at elastic deformation; without flow, $A = 2 \times 10^4$ MPa (thin curves) and $q = 2$ millions m$^3$/day, $A = 4 \times 10^4$ MPa (thick curves); $\sigma_1$ (full curve), $\sigma_0$ (broken curve), $\sigma_2$ (dotted curve), $p$ (chain curve); $S_0 = 10$ KPa, $k = 1$ Darcy, $p_s = 7.4$ MPa, $p_0 = 20.7$ MPa, $\nu = 0.3$, $\nu_e = 0.25$, $A = 2 \times 10^3$ MPa, $r_1 = 5.45$ cm, $A = 1.65$ cm, $h = 10$ m.
tions, the level of stresses in the sandstone is essentially reduced, and also difference between radial and angular stresses is reduced by 3 times near the wall of the filter.

The third important physical result for a case of elastic deformation in a vicinity of a stressed filter is that the stressed-state solution weakly depends on the permeability of the reservoir rock. A decrease of permeability by 2.5 does not essentially affect the elastic stress solution in a vicinity of SF; and, does not lead to an occurrence of a plastic deformation region because of the increase of flowing loading (Fig. 5).

Thus, the only elastic deformation of reservoir in the vicinity of the stressed filter is typical for UGS. The absence of a plastic region means that the reservoir is stable, including poorly consolidated sands. But for change of conditions to rather high values of Young’s modulus of a sandstone, a narrow zone of plastic deformation is formed around the SF.

5. Plastic deformation in a vicinity of stressed filter

Begin by finding a boundary condition on the wall of the SF with plastic deformation of the adjoining sandstone. First, consider a case where the plastic region (similar to that arising in a vicinity of uncased wellbore) also consists of an inner plastic zone where angular stress increases with the radius, and an outer plastic zone where angular stress decreases with removal from a filter axis (Risnes et al., 1982; Pyatakhin et al., 2003). In the inner plastic zone, vertical and angular
Where the plastic flow function is equal to Risnes et al.’s law, the elastic components of vertical and angular strains should coincide:

\[ \varepsilon_{\theta}^e = \frac{\sigma_{\theta} - v(\sigma_{r} + \sigma_{z}) - (1 - 2v)\beta p}{E} = \varepsilon_{z}^e \]  

The equality of plastic components of total strain follows from the associative law of flow (Risnes et al., 1982):

\[ \varepsilon_{\theta}^p = \frac{1}{l} \frac{\partial f}{\partial \sigma_{\theta}} = l \frac{\partial f}{\partial \sigma_{z}} = \varepsilon_{z}^p. \]  

Where the plastic flow function is

\[ f = \sigma_{z} - \sigma_{t} \tan^2 \alpha + (\tan^2 \alpha - 1) p - 2S_0 \tan \alpha = 0, \]  

and \( l \) is a scalar.

Thus, for an inner plastic zone the total vertical and angular strains are equal:

\[ \varepsilon_{\theta} = \varepsilon_{\theta}^e + \varepsilon_{\theta}^p = \varepsilon_{z} \]  

The displacement in an inner plastic zone is equal to: \( s = \varepsilon_{\theta} r \). The expression for displacement of an element of the filter on the external border was obtained above. Using a continuity of displacement on the filter border with the inner plastic zone at \( r = r_1 \) yields a boundary condition for plastic deformation of the rock around the filter:

\[ a \left( \frac{p_0 - p_2}{1 - v} + \frac{2r_0^2}{b} + 1 - v_t \right) = \frac{r_1^2 - r_0^2}{b} - v_t \]  

At plastic deformations of adjacent sandstone the SF applies compressive radial stress described by Eq. (26) to the rock.

The expressions for stress components in inner plastic zone was obtained by Pyatakhin et al. (2003):

\[ \sigma_{r} = p(r) + \frac{1}{t} \left[ \left( \frac{r}{r_1} \right)^t - 1 \right] \left( 2S_0 \tan \alpha - A \right) - \frac{B}{t + 1} \]  

\[ \times \left[ \left( \frac{r}{r_1} \right)^t - 1 \right] \left( \sigma_{r} - p_1 \right) \left( \frac{r}{r_1} \right)^t \]  

\[ \sigma_{z} = \sigma_{t} = p(r) + A - B \left[ \left( \frac{r}{r_1} \right)^t - 1 \right] \]  

\[ \times \left[ (t + 1) \left( \sigma_{t} - p_1 \right) + \left( 2S_0 \tan \alpha - A \right) \frac{1}{t} \right] \]  

where \( t = \tan^2 \alpha - 1 \).

Together with the formula for total radial stress (\( \sigma_{r_0} \), Eq. (26)) and that described by Pyatakhin et al. (2003) solutions for outer plastic zone and elastic region, the above expressions give the full stressed-state solution for bed in a vicinity of stressed filter.

Consider separately a case where the SF joins the outer plastic zone, and an inner plastic zone is absent. As the analysis shows, this case is typical of the stressed filters and is not typical of an uncased wellbore (Risnes et al., 1982; Pyatakhin et al., 2003). The equations obtained by Pyatakhin et al. (2003) is used for the stress solution in plastic zone. The radial stress is:

\[ \sigma_{r} = \left\{ \begin{align*} &c_1 r^{\gamma - 1} (a_3 + \gamma) + c_2 r^{\gamma - 1} (a_3 - \gamma) + a_1 \ln r \frac{1}{1 - \gamma^2} (1 + a_3) - a_3 a_8 \gamma - \frac{a_2 r^2}{\gamma^2 r_2} + a_7 (1 + a_3) \left\} / a_2 \right. \]  

where the designations are listed in Pyatakhin et al. (2003). Substituting \( r = r_1 \), on the border of the filter and rock, \( \sigma_{r_0} \) is obtained. The angular and vertical components of stress are found from the balance condition and Coulomb failure criterion:

\[ \sigma_{\theta} = \sigma_{t} + \frac{d \sigma_{r}}{dr}, \]  

\[ \sigma_{z} = \sigma_{t} \tan^2 \alpha - (\tan^2 \alpha - 1) p + 2S_0 \tan \alpha. \]  

The following expressions are used to determine (1) the unknown \( \sigma_{r_0} \), (2) the constants \( C_1 \) and \( C_2 \), (3) the radius of plastic/elastic boundary, \( r_{c} \), and (4) the total radial stress on this border \( \sigma_{r_c} \):

(1) The condition for a continuity of strain on the filter border with the rock:

\[ C_1 r_1^{\gamma - 1} + C_2 r_1^{\gamma - 1} + \alpha \gamma = \frac{a_8}{\gamma^2 r_1} + \frac{a_2 \ln r_1}{1 - \gamma^2} = \frac{1}{E_t} \]  

\[ \left[ \sigma_{r_0} (r_1^2 + r_0^2) - 2p r_0^2 \right] - v_t \sigma_{r_0} - (1 - v_t) p_1 \]  

(2) The expression for \( \sigma_{r_c} \) is obtained by substitution \( r = r_c \) in Eq. (29) for \( \sigma_{r} \).

(3) The condition for a continuity of vertical stress at the border of the plastic and elastic region of a rock at \( r = r_c \):

\[ \sigma_{r_c} \tan^2 \alpha - (\tan^2 \alpha - 1) p(r_c) + 2S_0 \tan \alpha = p_0 + \beta \frac{1 - 2v}{1 - v} [p(r_c) - p_2] \]
(4) Equality of displacement on this border at \( r = r_c \):

\[
C_1 \frac{r^{2\gamma-1}}{r_c} + C_2 \frac{r^{\gamma-1}}{r_c} - \alpha \gamma \frac{x_8}{\gamma^2 r_1} \frac{r}{C_1 r_c} + \frac{x_7 \ln r_c}{1 - \gamma} \]

\[
= \frac{1}{2G} \left[ -\sigma_{rc} + \frac{(\lambda + G)}{\lambda} \frac{\nu p_2 - G \beta A}{1 - \nu} \right] - \frac{\lambda}{\lambda + 2v} (p_0 - \beta p_2) - \frac{G \beta B}{2(\lambda + 2G)} \frac{G \beta p(r_c)}{1 - \nu} - \frac{\beta}{2(\lambda + 2G)} \left[ \frac{p(r_c) - A}{2} - \frac{B}{r_c} \right] \]

where \( \lambda \) and \( G \) are Lamé parameters of the rock. Together with the expression for \( \sigma_{r_0} \) five equations with five unknown values result, and this set of equations is reduced to two nonlinear equations for \( \sigma_{r_0} \) and \( r_c \):

\[
F_1(\sigma_{r_0}, r_c) = 0 \quad (35)
\]

\[
F_2(\sigma_{r_0}, r_c) = 0 \quad (36)
\]

The solutions of Eqs. (35) and (36), together with the formulas for stress components (Eqs. (29)–(31)), give the spatial stress distribution in a vicinity of a wellbore stressed filter surrounded by an outer plastic zone.

The mathematical models of the elastic and plastic deformation of a reservoir, in a vicinity of SF, were used in programs for computer calculations.

The analyses of the results show that for UGS conditions and a wire-wrapped SF, in absence of a fluid flow and at sufficiently high Young’s modulus of a sandstone, a narrow outer plastic zone is formed near to the filter (Figs. 6,7). A complete region of the plastic deformation consisting of inner and outer zones is not formed, in contrast to the case without SF (Risnes et al., 1982; Pyatakhin et al., 2003).

From the figures, even considering the plastic deformation, the region of the changed properties of a bed is only 15–20 cm from the axis of the filter. The region of the changed properties is basically in gravel if an external gravel pack used. For example, the external radius of the gravel for the wire-wrapped filter is about 15 cm (Kazaryan et al., 1998). This justifies the assumption of the identity of mechanical and fluid flow properties of a sandstone and gravel.

In Fig. 6, spatial distribution of stresses is given without fluid flow for two wire-wrapped SF (Kazaryan et al., 1998). Typical initial data is used (Table 1). It is seen that a narrow zone of plastic deformation with width 1–1.5 cm surrounds both filters, and this zone is slightly wider for the filter having a greater external diameter.

As already mentioned, the plastic region is absent for a wellbore stressed filter at the sandstone stratum with Young’s modulus equal to \( 2 \times 10^4 \) MPa, even without a fluid flow (Fig. 3). The boundary value of Young’s modulus for plastic region is \( 2.8 \times 10^4 \) MPa. From the literature (Nikolaevskiy, 1996) the greatest \( \bar{A} = 7 \times 10^4 \) MPa and the least values \( 2 \times 10^3 \) MPa are known for sandstones. The results of the appropriate...
stressed-state calculations for SF are given in Fig. 7 at production rate 1 million m³/day. Comparison with the case of $A_r=4*10^4$ MPa (Zheltov, 1966) (Fig. 6) shows that an increase in the Young’s modulus does not lead to a significant difference of results and an appreciable expansion of the plastic zone near the stressed filter. It is important to note that at low Young’s moduli of sandstones, the wire-wrapped SF does not disturb the initial stressed state of adjacent sandstone stratum. Deformation of sandstone in a vicinity of stressed filters at rather low values of Young’s modulus ($b<2.8*10^4$ MPa) is elastic (Fig. 7).

Thus, for the wire-wrapped SF at relatively high Young’s moduli of a sandstone close to filter, a narrow zone of plastic deformation is formed. Production of a complete plastic region perturbing the stability of a reservoir (as in the uncased wellbore) in the vicinity of a stressed filter does not occur.

6. First mechanism of a reservoir failure in a vicinity of wellbore stressed filter

The first critical condition is when the zone of plastic deformation starts to expand in the reservoir, i.e. when all the bed yields (Bratli and Risnes, 1981; Risnes et al., 1982; Pyatakhin et al., 2003; Pyatakhin and Kazaryan, 2004). The following formula for the critical production rate in the wellbore with a cylindrical stressed filter is obtained by taking into account the nonlinearity of the fluid flow:

$$q_c = \frac{(t+1)r_1\pi h\mu}{tk_bD\rho_a} \times \left\{ \frac{1+\frac{4(2S_0\tan t + t(\sigma_{10}-p_1))p_2k^2\beta_D\rho_aT_0}{p_a[(t+1)\mu^2T]^2}}{1} \right\}^{1/2}$$

Eq. (37) differs from that derived earlier for an uncased wellbore (Pyatakhin et al., 2003): a radicand factor $2S_0\tan t$ is replaced by $2S_0\tan t + t(\sigma_{10}-p_1)$. The effective stress $\sigma_{10}-p_1$ of a wire-wrapped SF (total radial stress minus bottom-hole pressure) is 4.5 MPa for initial data listed in the Table 1. Values of the sandstone cohesive strength do not exceed 0.1 MPa (Zotov et al., 1987). Therefore $2S_0\tan t \ll t(\sigma_{10}-p_1)$, thus in the formula for the critical production rate (Eq. (37)) the term $2S_0\tan t$ can be neglected.

The expression for the critical production rate of wire-wrapped stressed filters is:

$$q_c = \frac{(t+1)r_1\pi h\mu}{tk_bD\rho_a} \times \left\{ \frac{1+\frac{4\sigma_1^2+p_2k^2\beta_D\rho_aT_0}{p_a[(t+1)\mu^2T]^2}}{1} \right\}^{1/2}$$

Fig. 7. Radial stress (full curve), tangential stress (broken curve), and vertical stress (dotted curve) at production rate $q=1$ million m³/day, $k=1$ Darcy; $A_r=7*10^4$ MPa (thin curve) and $A_r=2*10^4$ MPa (thick curve); $h=10$ m, $p_2=7.4$ MPa, $p_0=20.7$ MPa, $r_1=5.45$ cm, $r_2=1.65$ cm.
Critical production rate and stability of bed in the vicinity of the stressed filter for UGS conditions do not depend on cohesive strength of the sand or sandstone surrounding the filter. This is the first basic result of the wellbore SF action. For an uncased wellbore (Risnes et al., 1982; Pyatakhin et al., 2003) or for perforated openings (Bratli and Risnes, 1981; Pyatakhin and Kazaryan, 2004) the stability of the bed strongly depends on the sandstone cohesive strength.

The results of modeling calculations for a poorly consolidated sandstone are given in Fig. 8 where (1) external radius of the filter \( r_1 = 10 \) cm, (2) effective radial stress \( \sigma_0 - p_1 = 0.5 \) MPa and other initial data are listed in Table 1. Calculations show that with an increasing production rate the narrow plastic region in a vicinity of the filter slightly extends. The bed failure, according to the first mechanism, should occur when there is a very high production rate (\( q_c = 5.2 \) millions m\(^3\)/day) and at enough low effective compressive radial stress of the filter. For UGS and a wire-wrapped stressed filters, the effective compressive radial stress of the filter is much greater (4.5 MPa) and the calculated critical production rates considerably exceed the achievable well production rates. It means that the second (and the main result) of the action of stressed filters is the stabilization of a reservoir. The first mechanism of bed destruction by shear stress does not work for conditions of UGS and a wire-wrapped SF. Thus, the stressed filter provides reservoir stability irrespective of cohesive strength of a rock adjoining the filter.

7. The absence of second mechanism of bed failure in a vicinity of the stressed filter

The possibility of formation of cracks near a cylindrical uncased wellbore (because of action of effective tensile stress) was discussed earlier (Risnes et al., 1982; Van den Hoek et al., 1996; Pyatakhin et al., 2003), and for perforated openings (Bratli and Risnes, 1981; Pyatakhin and Kazaryan, 2004). Destruction may occur when pressure in the bed increases radially faster than the radial stress and exceeds values of radial stress enough to fail. This is labeled the second failure mechanism of bed. The calculation results show that the radial stress for UGS appreciably exceeds the pressure of a fluid (Figs. 3,4). Radial stress on a wall of the filter exceeds bottom-hole pressure by approximately 4.5 MPa for wire-wrapped SF. This means that there are no effective tensile stresses for stressed filters with sufficient compressive radial stress. Neither the first nor the second mechanisms of bed failure occur when wellbore stressed filters are used.

8. Injection of fluid into bed

All derived formulas are valid for production and injection of a fluids in a reservoir. The computer programs also allow calculation of stresses in the vicinity of a SF for injection of fluid. Examples of calculation of stresses near the filter for production, injection and in the absence of fluid flow using specified earlier parameters are shown in Fig. 9. Fluid flow at sufficiently

![Fig. 8. Radial stress (full curve), tangential stress (broken curve), vertical stress (dotted curve), and fluid pressure (chain curve) at a production rate \( q = 5.18 \) millions m\(^3\)/day, close to critical (thick curve) and at \( q = 2 \) millions m\(^3\)/day (thin curve); effective filter reaction \( \sigma_0 - p_1 = 0.5 \) MPa at \( h = 10 \) m, \( p_2 = 7.4 \) MPa, \( p_0 = 20.7 \) MPa, \( v = 0.3 \), \( v_r = 0.25 \), \( A = 4 \times 10^4 \) MPa, \( A_f = 2 \times 10^5 \) MPa, \( S_0 = 10 \) KPa; \( r_1 = 5.45 \) cm, \( A = 1.65 \) cm, \( k = 1 \) Darcy.]
high production rates does not essentially disturb the stressed state in a vicinity of the wellbore SF. This result is important for underground gas storage where fluid flow changes direction because of cyclic operation.

9. Stresses and stability of a reservoir in a vicinity of SF at the presence of water

The flow becomes two-phase for simultaneous production of water and gas. For Forchheimer equation it is only necessary to use the values of the parameters that influence the second phase. Liu et al. (1995) proposed a formula for calculation of the non-Darcy flow coefficient for two-phase flow. Thus, the results obtained above for the elastoplastic deformation and stability of the reservoir near the stressed filters are transferred directly to joint flow of water and gas, during both production and injection of gas.

The empirical formulas obtained by Chen-Chjun-Syan (1962) may be used for relative permeabilities of the phases. Analysis has shown that phase permeability for gas at 50% water-saturation decreases by about 7 times; and at 20% content of water, by 1.5 times. At a high production rate \( q = 2 \text{ million m}^3/\text{day} \) reduction of permeability by 2.5 times does not essentially change the stressed state in a vicinity of wellbore SF (Fig. 4). Results of calculations are given at a fluid production with of 1 million m\(^3\)/day and in-place per-
meability $k = 1$ and 0.14 Darcy (Fig. 10). The narrow zone of plastic deformation disappears near the filter (Fig. 10) and entire bed is elastic when the permeability of the rock is reduced. The opposite conditions were observed for perforated openings (Bratli and Risnes, 1981; Pyatakhin and Kazaryan, 2004) and uncased wellbore (Risnes et al., 1982; Pyatakhin et al., 2003). A significant (~7 times) decrease of permeability varies the distribution of fluid pressure in the bed, but not the distribution of stresses in a vicinity of the SF. Thus stressed filters provide stability to the reservoir even for reduced relative permeability of the fluids with an increase of water-saturation.

The destruction of a low cohesive strength sandstone in an uncased well caused by reduction of coupling capillary forces was discussed by Pyatakhin et al. (2003). Calculations showed that in the vicinity of stressed filters a region of rock elastic deformation is formed for poorly consolidated sandstones with $S_0 = 10$ KPa (Figs. 3,4) and even for unconsolidated sand with $S_0 = 0$ (Fig. 2). In some cases near the filter a narrow zone of plastic deformation arises (Figs. 6,7,9,10). In both cases, with a wellbore SF, neither the first nor the second mechanisms of the reservoir destruction (by the shear and tensile stresses) are effective.

10. Conclusion

The SF stabilizes a poorly consolidated reservoir. In the vicinity of the stressed filter (at rather low reservoir Young’s modulus) an area of elastic deformation is formed. At a rather high value of Young’s modulus for a sandstone and close to wellbore SF, a narrow plastic zone forms; it does not disturb the stability of a bed. The mechanisms of reservoir destruction by shear (typical of perforated openings and uncased wellbores) and tensile (typical of perforated openings) stresses do not occur in conditions of underground storages of gas and use of SF with the parameters similar to wire-wrapped filters. Even in case of significant rock permeability reduction, the stressed filters stabilize the bed.

The theory that was developed can be used at design SF and wellbore completion for gas and oil fields, and underground storages of gas. For UGS, production of a fluid is periodically replaced by its injection into the reservoir. Thus fluid flow changes direction. Stabilizing action of the stressed filter is shown for both fluid injection and production. Flow, even at a production rate of 1 million m$^3$/day, does not disturb the stressed state in a vicinity of SF. Wellbore stressed filters stabilize the bed during simultaneous product of water and gas. Even at high production rates, reduction of relative permeability to the hydrocarbon phase (and cohesion with increase of water content) does not result to rock failure in a vicinity of SF.

Thus stressed filters have the ability to prevent movement of sand and they protect the bed from destruction during fluid production and injection. They represent effective tools for control of sand production for gas and oil wells at unstable reservoir conditions.

Nomenclature

\[ A \quad \text{constant given by Eq. (3)} \]
\[ A_\text{F} \quad \text{constant given by Eq. (4)} \]
\[ a \quad \text{constant given by Eq. (17)} \]
\[ a_1, a_2, a_3, a_4, a_7, \tilde{a}_7, a_8 \quad \text{constants in Eq. (29)} \]
\[ B \quad \text{constant given by Eq. (3)} \]
\[ B_\text{F} \quad \text{constant given by Eq. (4)} \]
\[ b \quad \text{constant given by Eq. (17)} \]
\[ C_1, C_2 \quad \text{constants in Eq. (29)} \]
\[ c_1, c_2 \quad \text{integration constants in the displacement solution (Eq. (6))} \]
\[ \tilde{c}_1, \tilde{c}_2 \quad \text{integration constants in the elastic solution (Eq. (8))} \]
\[ E \quad \text{Young’s modulus of the rock (m/Lt}^2\text{), MPa} \]
\[ E_f \quad \text{Young’s modulus of the filter (m/Lt}^2\text{), MPa} \]
\[ f \quad \text{plastic flow function} \]
\[ G \quad \text{Lamé parameter of the rock (m/Lt}^2\text{), MPa} \]
\[ G_f \quad \text{Lamé parameter of the filter (m/Lt}^2\text{), MPa} \]
\[ g \quad \text{acceleration of gravity (L/t}^2\text{), m/s}^2 \]
\[ h \quad \text{reservoir thickness (L), m} \]
\[ k \quad \text{permeability (L}^2\text{), Darcy units (1.02} \times 10^{-12}\text{ m}^2\text{)} \]
\[ k_f \quad \text{hydraulic conductivity (L/t), m/s} \]
\[ l \quad \text{a scalar} \]
\[ p \quad \text{fluid pressure (m/Lt}^2\text{), MPa} \]
\[ p_0 \quad \text{overburden pressure (m/Lt}^2\text{), MPa} \]
\[ p_1 \quad \text{fluid bottom-hole pressure (m/Lt}^2\text{), MPa} \]
\[ p_2 \quad \text{fluid pressure at outer boundary (m/Lt}^2\text{), MPa} \]
\[ p_{\text{at}} \quad \text{atmospheric pressure (m/Lt}^2\text{), MPa} \]
\[ q \quad \text{wellbore production rate (L}^3\text{/t), millions m}^3\text{/day} \]
\[ q_c \quad \text{wellbore critical production rate (L}^3\text{/t), millions m}^3\text{/day} \]
\[ r \quad \text{radial distance from center of wellbore (L), m} \]
\[ r_0 \quad \text{internal filter radius (L), cm} \]
\[ r_1 \quad \text{wellbore radius and external radius of the filter (L), cm} \]
\[ r_2 \quad \text{outer boundary radius (L), m} \]
\[ r_c \quad \text{radius of plastic/elastic boundary (L), cm} \]
\[ s \quad \text{radial displacement (L), mm} \]
\[ S_0 \quad \text{cohesive strength (m/Lt}^2\text{), Kpa} \]
SF stressed filter

\( T \) reservoir temperature (T), K

\( T_0 \) 293 K, temperature (T)

\( t \) \( \tan^2 \alpha - 1 \), constant

UGS underground gas storage

\( z \) coefficient applied to ideal gas compressibility to account for non-linear behavior at high pressures

\( \alpha \) failure angle, degrees

\( 1 - \beta \) ratio of a material and volume compressibilities of the rock

\( 1 - \beta_f \) ratio of a material and volume compressibilities of the filter

\( \beta_D \) non-Darcy flow coefficient of the Forchheimer equation

\( \nu \) constant in Eq. (29)

\( \delta \) thickness of a wall of the filter (L, cm)

\( \varepsilon_r, \varepsilon_0, \varepsilon_z \) total strain components, compressive positive (\( \Delta \), L/L)

\( \varepsilon_r^e, \varepsilon_0^e, \varepsilon_z^e \) elastic strain components, compressive positive (\( \Delta \), L/L)

\( \varepsilon_r^p, \varepsilon_0^p, \varepsilon_z^p \) plastic strain components, compressive positive (\( \Delta \), L/L)

\( \varepsilon_{z0} \) constant vertical total strain, compressive positive (\( \Delta \), L/L)

\( \varepsilon_{z0} \) constant vertical elastic strain, compressive positive (\( \Delta \), L/L)

\( \lambda \) Lamé parameter of the rock (m/Lt²), MPa

\( \lambda_f \) Lamé parameter of the filter (m/Lt²), MPa

\( \mu \) fluid viscosity (m/Lt), Pa s

\( \nu \) Poisson’s ratio of the rock

\( \nu_f \) Poisson’s ratio of the filter

\( \rho \) density of fluid (m/L³), kg/m³

\( \rho_{at} \) density of gas at atmospheric pressure (m/L³), kg/m³

\( \sigma_n, \sigma_\theta, \sigma_z \) total stress components in cylindrical coordinates (m/Lt²), MPa

\( \sigma_{r0} \) total radial stress at wellbore wall (m/Lt²), MPa

\( \sigma_{rc} \) total radial stress at plastic/elastic boundary (m/Lt²), MPa.
