Automatic control system of a rear-wheel drive vehicle moving on a sloped weak sandy terrain

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Abstract

The general mechanism of tractive performance of a four-wheel vehicle with rear-wheel drive moving up and down a sloped sandy soil has been considered theoretically. For the given vehicle dimensions and terrain-wheel system constants, the relationships among the effective tractive or braking effort of the vehicle, the amount of sinkage of the front and rear wheels, and the slip ratio were analysed by simulation. The optimum eccentricity of the vehicle’s center of gravity and the optimum application height of the drawbar-pull for obtaining the largest value of maximum effective tractive or braking effort could be calculated by means of the analytical simulation program. For a 5.88 kN weight vehicle, it was found that the optimum eccentricity of the center of gravity $e_{\text{opt}}$ was $1/6$ for the range of slope angle $0 \leq \beta \leq \pi/24$ rad during driving action of the rear wheel and $e_{\text{opt}}$ was also $1/6$ for the range of slope angle $-\pi/24 \leq \beta \leq 0$ rad during braking action of the rear wheel. The optimum application height $H_{\text{opt}}$ was found to be 35 cm for the range of slope angle $0 \leq \beta \leq \pi/24$ rad during driving action of the rear wheel and $H_{\text{opt}}$ was 0 cm for the range of slope angle $-\pi/24 \leq \beta \leq 0$ rad during braking action of the rear wheel. © 1998 ISTVS. All rights reserved.

Nomenclature

$a$: terrain-wheel system constant obtained from plate traction test (cm$^{-1}$)

$B_{f}$: width of front wheel (cm)

$B_{r}$: width of rear wheel (cm)

$c_{0}$: terrain-wheel system constant obtained from plate slip sinkage test (cm$^{2}c^{1}+c^{2}+1N^{-c^{1}}$)

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$c_1$: terrain-wheel system constant obtained from plate slip sinkage test
$c_2$: terrain-wheel system constant obtained from plate slip sinkage test
$D$: wheel base (cm)
$e$: eccentricity of center of gravity $G$ of vehicle
$eD$: amount of eccentricity of center of gravity $G$
$e_{fb}$: eccentricity of ground reaction acting on front wheel during braking action (cm)
$e_{opt}$: optimum eccentricity of center of gravity of vehicle
$e_{rb}$: eccentricity of ground reaction acting on rear wheel during braking action (cm)
$e_{rd}$: eccentricity of ground reaction acting on rear wheel during driving action (cm)
$F$: application point of effective tractive or braking effort of vehicle
$G$: point of center of gravity of vehicle
$h_g$: height of center of gravity $G$ of vehicle (cm)
$H$: height of application of effective tractive or braking effort of vehicle (cm)
$H_{opt}$: optimum application height of effective tractive or braking effort of vehicle (cm)
i: slip ratio or skid
$i_{fb}$: skid of front braking wheel
$i_{rb}$: skid of rear braking wheel
$i_{rd}$: slip ratio of rear driving wheel
$k_c$: terrain-wheel system constant obtained from plate loading test
\(\text{N cm}^{-(n+1)}\)
$k_{cr}$: terrain-wheel system constant obtained from plate unloading test
\(\text{N cm}^{-(n+1)}\)
$k_{cr}$: terrain-wheel system constant obtained from plate loading test
\(\text{N cm}^{-(n+2)}\)
$k_{фр}$: terrain-wheel system constant obtained from plate unloading test
\(\text{N cm}^{-(n+2)}\)
l_{fb}: height between front axle $O_f$ and application point of normal ground reaction acting on front braking wheel (cm)
l_{rb}: height between rear axle $O_r$ and application point of normal ground reaction acting on rear braking wheel (cm)
l_{rd}: height between rear axle $O_r$ and application point of normal ground reaction acting on rear driving wheel (cm)
$L$: distance between central axis of vehicle and point $F$ acting effective tractive or braking effort (cm)
$L_{фcb}$: compaction resistance acting on front wheel (kN)
$L_{rcb}$: compaction resistance acting on rear wheel (kN)
m_c: terrain-wheel system constant obtained from plate traction test (kPa)
m_r: terrain-wheel system constant obtained from plate traction test
$M_f$: bottom-dead-center of front wheel
$M_r$: bottom-dead-center of rear wheel
$n$: terrain-wheel system constant obtained from plate loading test
$n_r$: terrain-wheel system constant obtained from plate unloading test
$O_f$: point of axle of front wheel
$O_r$: point of axle of rear wheel
$Q_{rd}$: driving torque of rear wheel (kN cm)
$Q_{rd}/R_r$: tangential driving force acting on rear wheel (kN)
$Q_{rb}$: braking torque of rear wheel (kN cm)
$Q_{rb}/R_r$: tangential braking force acting on rear wheel (kN)
$R_f$: radius of front wheel (cm)
$R_r$: radius of rear wheel (cm)
$R_f\omega_f$: circumferential speed of front wheel (cm s$^{-1}$)
$R_r\omega_r$: circumferential speed of rear wheel (cm s$^{-1}$)
$s$: rut depth of vehicle (cm)
$S_f$: amount of sinkage of front wheel (cm)
$S_r$: amount of sinkage of rear wheel (cm)
$T$: effective tractive or braking effort of vehicle (kN)
$T_B$: effective braking effort of vehicle (kN)
$T_{B-20}$: effective braking effort of vehicle at skid of $-20\%$ (kN)
$T_{fb}$: effective braking force of front wheel (kN)
$T_{fd}$: effective driving force of front wheel (kN)
$T_{D}$: effective tractive effort of vehicle (kN)
$T_{D_{max}}$: maximum tractive effort of vehicle (kN)
$T_{rd}$: effective driving force of rear wheel (kN)
$T_{rb}$: effective braking force of rear wheel (kN)
$u_f$: amount of rebound of front wheel (cm)
$u_r$: amount of rebound of rear wheel (cm)
$V$: vehicle speed (cm s$^{-1}$)
$V_0$: loading speed at plate loading test (cm s$^{-1}$)
$W$: vehicle weight (kN)
$W_f$: front axle load (kN)
$W_r$: rear axle load (kN)
$\beta$: slope angle of terrain (rad)
$\beta_{max}$: maximum slope angle of terrain (rad)
$\theta$: angle of inclination of vehicle (rad)
$\theta_{reb}$: central angle between point acting ground reaction and bottom-dead-center $M_f$ of front braking wheel (rad)
$\theta_{rad}$: central angle between point acting ground reaction and bottom-dead-center $M_r$ of rear driving wheel (rad)
$\theta_{reb}$: entry angle of rear braking wheel (rad)
$\kappa$: terrain-wheel system constant obtained from plate loading test
$\lambda$: terrain-wheel system constant obtained from plate unloading test
$\omega_f$: angular velocity of front wheel (rad s$^{-1}$)
$\omega_r$: angular velocity of rear wheel (rad s$^{-1}$)
1. Introduction

For the automation of wheeled vehicles of construction machinery used for earth moving, it is considered very important that the eccentricity of center of gravity of the vehicle and the application height of the drawbar-pull should be controlled automatically to the optimum values for obtaining the largest effective tractive effort when the vehicle is towing other construction machinery uphill on a sloped terrain, or for obtaining the largest effective braking effort when the vehicle is carrying downhill.

The objective of this paper was to establish a control system for the position of the eccentricity of center of gravity and the application height of the tractive effort that would optimize the tractive performance of a wheeled vehicle.

For this paper, the tractive performance of a 5.88 kN weight, two axle, four-wheel vehicle with rear-wheel drive and brake system moving uphill during driving action and downhill during braking action on a sloped decomposed granite soil was considered theoretically. The tractive performance of the wheeled vehicle, constructed by two front wheels during unpowered rolling state and two rear wheels during driving or braking action, was simulated for given vehicle dimensions and terrain-wheel system constants. The relationship among the effective tractive or negative braking effort of the vehicle, the effective driving or negative braking force of the rear wheel, the amount of sinkage of the front and rear wheels, and the slip ratio of the rear wheel and skid was analysed using the simulation program. Based on the simulation analysis’ results regarding the effect of the position of the eccentricity of center of gravity and the application height on the tractive performance of the wheeled vehicle, the optimum eccentricity of center of gravity and the optimum application height for obtaining the largest effective tractive effort or the largest absolute value of negative effective braking effort of the vehicle could be determined. The effect of the slope angle on the optimum eccentricity of center of gravity and the optimum application height was also investigated.

Finally, some automatic control systems using an inclinometer sensor for maintaining the position of the optimum eccentricity of center of gravity and the optimum application height for various terrain conditions and slope angles are presented.

2. Simulation analysis

The tractive performance of a two-axle, four wheel vehicle with rear wheel drive and brake system moving up or down a sloped terrain of loose air-dried decomposed granite was considered as the combined effect of the effective braking force in the unpowered rolling state of the front wheel and the effective driving force [1] during driving action, or the effective braking force during braking action [2] of the rear wheel [3].

Fig. 1 shows the vehicle dimensions and several forces acting on the rear-wheel drive vehicle moving up sloped terrain of angle $\beta (\geq 0)$ during driving action. Fig. 2
shows several forces acting on the rear-wheel brake vehicle moving down sloped terrain of angle $\beta$ during braking action. The vehicle weight $W$ acts vertically on the vehicle’s center of gravity $G$, and it is divided into the normal component $W \cos \beta$ and the tangential component $W \sin \beta$ to the sloped terrain. $W \sin \beta$ is defined as a slope resistance due to the vehicle weight. The front axle load $W_f$ and the rear axle load $W_r$ act normally to the terrain surface. The slope resistance $W_f \tan \beta$ and $W_r \tan \beta$ act parallel to the terrain surface, on the front axle $O_f$ and the rear axle $O_r$, respectively. The position of the center of gravity $G$ is located at a height $h_g$ perpendicular to the line $O_fO_r$ and at a distance of the amount of eccentricity $eD$ ($e$ is defined as the eccentricity of center of gravity of the vehicle) from the central axis of the vehicle. $D$ is the wheel base. $R_f$ and $R_r$ are the radii of front and rear wheel respectively. $B_f$ and $B_r$ are the widths of front and rear wheel respectively. The driving torque $O_{rd}$ or the braking torque $Q_{rb}$ acts around the rear axle $O_r$. The position of the application point $F$ of the effective tractive effort $T_D$ or the effective braking effort $T_B$ is located at a distance $L$ from the central axis of the vehicle and at an application height $H$ from the line $O_fO_r$. During driving action of the rear wheel, the effective braking force $T_{rb}$ in unpowered rolling state is defined to act on the front axle $O_f$ parallel to the terrain surface. The effective driving force $T_{rd}$ is defined as acting on the rear axle $O_r$ parallel to the terrain surface, as shown in Fig. 1. During braking action of the rear wheel, the effective braking force $T_{rb}$ in unpowered rolling state is defined as acting on the front axle $O_f$ parallel to the terrain surface. The effective braking force $T_{rb}$ is defined to act on the rear axle $O_r$ parallel to the terrain surface, as shown in Fig. 2. The vehicle’s angle of inclination $\theta_i$ is calculated as the angle between the line $O_fO_r$ and the terrain surface. For the amount of sinkage $s_f$ at the bottom-dead-center $M_f$ of the front wheel and for the amount of sinkage $s_r$ at the bottom-dead-center $M_r$ of the rear wheel, the vehicle’s angle of inclination of $\theta_i$ can be calculated as,
where $u_f$ is the amount of rebound of the terrain at the front wheel.

2.1. Rear-wheel drive vehicle

During driving action of the rear wheel, the unpowered rolling resistance (i.e. the compaction resistance) $L_{fcb}$ parallel to the terrain surface and the normal ground reaction $W_f$ are defined to act on the forward contact part of the front wheel at a distance of the amount of eccentricity $e_{fb} = R_f \sin \theta_{fcb}$ and $l_{fb} = R_f \cos \theta_{fcb}$. The compaction resistance $L_{red}$ parallel to the terrain surface, the tangential driving force $Q_{rd} = R_r$, and the normal ground reaction $W_r - (Q_{rd}/R_r) \sin \theta_{red}$ are defined to act on the forward contact part of the rear wheel at the distance of the amount of eccentricity $e_{rd} = R_r \sin \theta_{red}$ and $l_{rd} = R_r \cos \theta_{red}$ (Fig. 1).

For the vehicle speed $V$, the angular velocities $\omega_f$ and $\omega_r$ of the front and rear wheel, the skid $i_{fb}$ of the front wheel and the slip ratio $i_{rd}$ of the rear wheel are expressed as follows;

$$i_{fb} = \frac{R_f \omega_f}{V} - 1$$  \hspace{1cm} (2)

$$i_{rd} = 1 - \frac{V}{R_r \omega_r}$$  \hspace{1cm} (3)

From the force equilibrium in parallel and normal directions to the terrain surface,

$$T_{rd} = \frac{Q_{rd}}{R_r} \cos \theta_{red} - L_{red} - W_r \tan \beta$$  \hspace{1cm} (4)

$$T_{fb} = -L_{fcb} - W_f \tan \beta$$  \hspace{1cm} (5)
are obtained.

From the momental equilibrium around the rear axle \(O_r\),

\[
    W_{f}D\cos\theta_{t} + L_{fcb}D\sin\theta_{t} - W\cos\beta \left\{ \frac{D}{2} - (eD + h_g\tan\theta_{t}) \right\} \cos\theta_{t}
    + HT_D\cos\theta_{t} - \left( L - \frac{D}{2} \right) T_{D}\sin\theta_{t} + W\sin\beta \left[ \frac{h_g}{\cos\theta_{t}} \right]
    + \left\{ \frac{D}{2} - (eD + h_g\tan\theta_{t}) \right\} \sin\theta_{t} = 0
\]

is obtained. Here, \(L_{fcb}\) is considered to act on the front axle \(O_f\). The effective tractive effort \(T_{D}\) can be calculated from the compaction resistance \(L_{fcb}\), the effective driving force \(T_{rd}\) and the compaction resistance \(L_{fcb}\), the effective driving force \(T_{rd}\) and the slope resistance. The relationships between \(T_{D}\) and \(i_{rd}\), \(T_{rd}\) and \(i_{rd}\), \(Q_{rd}\) and \(i_{rd}\) and between \(s_f\), \(s_r\) and \(i_{rd}\) were determined by the analytical simulation program, of which a flow chart is shown in Fig. 3.

2.2. Rear-wheel brake vehicle

During braking action of the rear wheel the compaction resistance \(L_{fcb}\) parallel to the terrain surface and the normal ground reaction \(W_{f}\) are defined to act on the forward contact part of the front wheel at the distance of the amount of eccentricity \(e_{fb} = R_f\sin\theta_{fcb}\) and \(l_{fb} = R_f\cos\theta_{fcb}\). The compaction resistance \(L_{red}\) parallel to the terrain surface, the tangential braking force \(Q_{rb}/R_r\), and the normal ground reaction \(W_{r} - (Q_{rb}/R_r)\sin\theta_{reb}\) are defined to act on the forward contact part of the rear wheel at the distance of the amount of eccentricity \(e_{rb} = R_r\sin\theta_{reb}\) and \(l_{rb} = R_r\cos\theta_{reb}\) (Fig. 2).

For vehicle speed \(V\), the angular velocities \(\omega_f\) and \(\omega_r\) of the front and rear wheel, the skid \(i_{fb}\) of the front wheel and the skid \(i_{rb}\) of the rear wheel are expressed as follows:

\[
    i_{fb} = \frac{R_f\omega_f}{V} - 1
\]

\[
    i_{rb} = \frac{R_r\omega_r}{V} - 1
\]

From the force equilibrium in parallel and normal directions to the terrain surface,

\[
    T_{rb} = \frac{Q_{rb}}{R_r}\cos\theta_{reb} - L_{reb} - W_{r}\tan\beta
\]
Fig. 3. Flow chart for simulation analysis of RWD (B).
are obtained.

From the momental equilibrium around the rear axle $O_r$,

\[
W_f D \cos \theta_t + L_{fcb} D \sin \theta_t - W \cos \beta \left\{ \frac{D}{2} - (eD + h_g \tan \theta_t) \right\} \\
\cos \theta_t + HT_B \cos \theta_t - \left( L - \frac{D}{2} \right) T_B \sin \theta_t + W \sin \beta \left[ \frac{h_g}{\cos \theta_t} \right] + \left\{ \frac{D}{2} - (eD + h_g \tan \theta_t) \right\} \sin \theta_t \right] = 0
\]  

(15)

is obtained. Here, $L_{fcb}$ is considered to act on the front axle $O_f$. The effective braking effort $T_B$ can be calculated from the compaction resistance $L_{fcb}$, the effective braking force $T_{rb}$ and the slope resistance. The relationships between $T_B$ and $i_{rb}$, $T_{rb}$ and $i_{rb}$, $Q_{rb}$ and $i_{rb}$ and between $s_f$, $s_r$ and $i_{rb}$ were determined by the same analytical simulation program.

2.3. Analytical results

The vehicle’s tractive performance was simulated using the terrain-wheel system constants given in Table 1 and the vehicle dimensions given in Table 2. A soil sample of the air-dried decomposed granite soil with grain-size below 4.75 mm, dry density of 1.60 g cm$^{-3}$ and cone index of 430 kPa (measured by cone penetrometer of 5 cm height, 30° apex angle and 6.61 cm$^2$ base area) was used to measure the terrain-wheel system constants.

Fig. 4 shows the relationship among the driving or braking force $Q/R_f$ ($Q_{rd}/R_f$ or $Q_{rb}/R_f$) of the rear wheel, the effective tractive or braking effort $T$ ($T_D$ or $T_B$) of the vehicle running on terrain of slope angle $\beta = \pi/36$ or $-\pi/36$rad at the height of application force $H = 15$ cm and the slip ratio $i$ ($i_{rd}$ or $i_{rb}$) of the rear wheel. $Q/R_f$ is always larger than $T$ for the whole range of the slip ratio $i$ and $Q_{rd}/R_f$ increases with the increment of the slip ratio $i$. At the unpowered rolling state of the rear wheel, $Q/R_f = Q_{rb}/R_f$ comes to zero at $i = i_{rb} = -1.8\%$ for $\beta = -\pi/36$rad during braking action. At the self-propelling state of the vehicle, $T = T_D$ comes to zero at $i = i_{rd} = 7.5\%$ for $\beta = \pi/36$rad during driving action.

Fig. 5 shows the relationship among the amount of sinkage $s_f$ of the front wheel, $s_r$ of the rear wheel, $s$ of the vehicle and the slip ratio $i$ ($i_{rd}$ or $i_{rb}$) of the rear wheel. The total amount of sinkage $s$ of the vehicle (i.e. the rut depth) can be calculated as,

\[ s = s_f - u_f + s_r - u_r \]

(16)

where $u_r$ is the amount of rebound of the terrain at the rear wheel.
The eccentricity of the position of the vehicle’s center of gravity on the effective tractive or braking effort can be analysed when the two-axle, four-wheel vehicle with rear-wheel drive is operating during driving action on loose sandy sloped terrain while pulling other construction machinery. The optimum eccentricity of the center of gravity was defined as the eccentricity that obtains the largest value of the maximum effective tractive effort $T_{D_{\text{max}}}$ [4].
Here, the value of $e_{\text{opt}}$ should be determined within $-1/6 \leq e_{\text{opt}} \leq 1/6$ for the safe driving operation of the vehicle. The wheeled vehicle results in a dangerous running state due to the negative axle load of front or rear wheels when the resultant force of the vehicle weight and the effective tractive or braking effort ($T_D$ or $T_B$) applies at the outside range of the middle third of the wheel base.

Fig. 6(a) shows the relationship between the effective tractive effort $T_D$ and the slip ratio $i_{rd}$ of rear wheel for the eccentricity $e = -0.20, -0.10, 0.00, 0.10, 0.20$ and $0.30$ for $\beta = \pi/36$ rad and $H = 35$ cm. As shown in this figure, the maximum value of the effective tractive effort $T_{D_{\text{max}}}$ increases with the increment of eccentricity $e$. Only in the case of $e = 0.20$ and $0.30$, does the effective tractive effort show positive values. In the case of another eccentricity, the vehicle has to be pushed because of
the negative effective tractive effort. Fig. 6(b) shows the relationship among the maximum effective tractive effort $T_{D_{\text{max}}}$, the corresponding effective driving force $T_{rd}$ of rear wheel and the eccentricity $e$ of the vehicle’s center of gravity. $T_{D_{\text{max}}}$ increases noticeably with the increase of $e$ while $T_{rd}$ increases gradually with $e$. When $T_{D_{\text{max}}}$ shows negative values, the vehicle drops down and is unable to pull any more.

$T_{D_{\text{max}}}$ has its largest value 0.058 kN at $e = 1/6$. In this case, the skid $i_{fb}$ of the front wheel in unpowered rolling state is maintained from $-5.6\%$ to $-7.1\%$. So, it is clear that the optimum eccentricity, $e_{\text{opt}}$ is 1/6 for the rear-wheel drive vehicle moving uphill on terrain of slope angle $\beta = \pi/36$ rad.

### 3.2. Rear-wheel brake vehicle

The effect of the position of the vehicle’s center of gravity $eD$ on the effective braking effort $T_B$ can also be analysed when the two-axle, four-wheel vehicle with rear-wheel brake system is operating during braking action on the loose sandy sloped terrain while pushing other construction machinery. The optimum eccentricity of the center of gravity $e_{\text{opt}}$ was defined as the eccentricity that obtains the largest absolute value of the effective braking effort $|T_{B_{-20}}|$ at $i_{fb} = -20\%$, which is considered a reasonable skid to avoid an excessive wear phenomenon of a wheel.
Fig. 6. (a) Relationship between effective tractive effort ($T_D$) and slip ratio ($i_{rd}$) for various kinds of eccentricity ($e$). (b) Relationship among maximum effective tractive effort ($T_{D_{\text{max}}}$), effective driving force ($T_{rd}$) and eccentricity ($e$).
The value of $e_{\text{opt}}$ should be also determined within $-1/6 \leq e_{\text{opt}} \leq 1/6$ for the safe braking operation of the vehicle.

Fig. 7(a) shows the relationship between the effective braking effort $T_B$ and the skid $i_{rb}$ of the rear wheel for the eccentricity $e = 0.20, 0.10, 0.00, -0.10, -0.20$ and $-0.30$ for $\beta = -\pi/36 \text{rad}$ and $H = 35 \text{ cm}$. The effective braking effort $T_B$ decreases with the increment of the eccentricity $e$. Fig. 7(b) shows the relationship among the effective braking effort $T_{B-20}$, at $i_{rb} = -20\%$, the corresponding effective braking force $T_{rb}$ of rear wheel and the eccentricity $e$ of the vehicle’s center of gravity.

Both $T_{rb}$ and $T_{B-20}$ decrease noticeably with the increase of $e$, while $T_{rb}$ is always larger than $T_{B-20}$. The absolute value of the effective braking effort $|T_{B-20}|$ at $i_{rb} = -20\%$, reaches its largest value $1.144 \text{ kN}$ at $e = 1/6$. In this case, the skid $i_{rb}$ of the front wheel in unpowered rolling state is maintained from $-1.9\%$ to $1.1\%$. So, it is clear that the optimum eccentricity is determine as $e_{\text{opt}} = 1/6$ for the rear-wheel brake vehicle moving downhill on terrain of slope angle $\beta = -\pi/36 \text{rad}$.

4. Height of application force control system

The effect of the height of application force $H$ on the effective tractive effort $T_D$ can be analysed when the two-axle, four wheel vehicle with rear-wheel drive is operating during driving action on the loose sandy sloped terrain to pull other construction machinery. The optimum application height $H_{\text{opt}}$ was defined as the height of application force that obtains the largest value of the maximum effective tractive effort within $0 \leq H_{\text{opt}} \leq 35 \text{ cm}$, considering the vehicle clearance and the hook position for pulling other construction machinery.

Fig. 8(a) shows the relationship among the maximum effective tractive effort $T_{D_{\text{max}}}$ of the rear-wheel driving vehicle, the corresponding effective driving force $T_{rd}$ of the rear wheel and the application height $H$ of the effective tractive effort. These simulation results were obtained for the slope angle $\beta = \pi/36 \text{rad}$ and the eccentricity $e = 1/6$. As a result, the largest value of the maximum effective tractive effort $T_{D_{\text{max}}} = 0.048 \text{ kN}$ is obtained at the upper limit of $H = 35 \text{ cm}$. So, it is clear that the optimum application height is determined as $H_{\text{opt}} = 35 \text{ cm}$ for this rear-wheel drive vehicle moving uphill on terrain of slope angle $\beta = \pi/36 \text{rad}$.

On the other hand, the effect of the height of application force $H$ on the effective braking effort $T_B$ can also be analysed when the two-axle, four wheel vehicle with rear-wheel brake system is operating during braking action on the loose sandy sloped terrain to push other construction machinery. The optimum application height $H_{\text{opt}}$ was defined as the height of application force that obtains the largest absolute value of the effective braking effort $|T_{B-20}|$ at $i_{rb} = -20\%$ within $0 \leq H \leq 35 \text{ cm}$, for the same safety reason mentioned above.

Fig. 8(b) shows the relationship among the effective braking effort $T_{B-20}$ at $i_{rb} = -20\%$ of the rear-wheel brake vehicle, the corresponding effective braking force $T_{rb}$ of the rear wheel and the application height $H$ of the effective braking effort. These simulation results were obtained for the slope angle $\beta = -\pi/36 \text{rad}$ and the eccentricity $e = 1/6$. As a result, the largest absolute value of the effective braking
Fig. 7. (a) Relationship between effective braking force \( T_b \) and skid \( \dot{i}_{rb} \) for various kinds of eccentricity \( e \). (b) Relationship between effective braking effort \( T_{B-20} \), effective braking force \( T_{rb} \) and eccentricity \( e \).
Fig. 8. (a) Relationship among maximum effective tractive effort ($T_{D_{\text{max}}}$), effective driving force ($T_{\text{rd}}$) and application height ($H$). (b) Relationship among effective braking effort ($T_{B_{-20}}$), effective braking force ($T_{\text{rb}}$) and application height ($H$).
effort at $i_{rb} = -20\%$, $|T_{B-20}| = 1.472$ kN is obtained at $H = 0$ cm. So, it is clear that the optimum application height is determined as $H_{opt} = 0$ cm for this rear-wheel brake vehicle moving downhill on terrain of slope angle $\beta = -\pi/36$ rad.

5. Effect of slope angle

The effects of the slope angle $\beta$ of the terrain on the effective tractive effort $T_D$ and on the effective braking effort $T_B$ can be analysed when the two-axle, four wheel vehicle with rear-wheel drive and brake is operating during driving and braking action on the loose sandy sloped terrain to pull or push other construction machinery.

Fig. 9(a) shows the relationship between the effective tractive effort $T_D$ of the vehicle and slip ratio $i_{rd}$ of the rear wheel for various slope angles $\beta$ when the vehicle with rear-wheel drive is operating at the combination of $e_{opt} = 1/6$ and $H = 15$ cm. $T_D$ increases with the increase of $i_{rd}$, but it decreases remarkably with $i_{rd}$ due to the increasing amount of sinkage of the rear wheel and the increasing positive slope resistance $W\sin \beta$ after reaching the maximum value $T_{D\text{max}}$ at about $i_{rd} = 9 \sim 14\%$. The maximum effective tractive effort $T_{D\text{max}}$ decreases with the increase of the slope angle $\beta$.

Fig. 9(b) shows the relationship between the effective braking effort $T_B$ of the vehicle and skid $i_{rb}$ of the rear wheel for various slope angles $\beta$ when the vehicle with rear-wheel brake is operating at the combination of $e_{opt} = 1/6$ and $H = 15$ cm. In general, the effective braking effort $|T_{B-20}|$ at $i_{rb} = -20\%$ decreases with the increase of the slope angle $|\beta|$ due to the increasing negative slope resistance $W\sin \beta$.

5.1. Optimum eccentricity

Fig. 10(a) shows the relationship between the total amount of sinkage of the vehicle $s$ and the slip ratio $i_{rd}$ of the rear wheel for various slope angles $\beta$ when the vehicle with rear-wheel drive is operating at the values of $e_{opt}$ and $H$ mentioned above. For the same slip ratio, $s$ increases slightly with the increase of $\beta$. Fig. 10(b) shows the relationship between the total amount of sinkage of the vehicle $s$ and the skid $i_{rb}$ of the rear wheel for various slope angles $\beta$ when the vehicle with rear-wheel brake is operating at the same values of $e_{opt}$ and $H$ mentioned above. From these figures, it is clear that, for the same slip ratio $i_{rd} = |i_{rb}|$, $s$ during the driving action of the rear wheel becomes larger than that during the braking action of the rear wheel due to the increasing amount of slippage of the rear wheel during driving action [5].

Fig. 11(a) shows the relationship among the maximum effective tractive effort $T_{D\text{max}}$, the corresponding effective driving force $T_{rd}$ and the slope angle $\beta$ of the terrain for the eccentricity $e = -1/6$, 0 and 1/6 when the vehicle with rear-wheel drive is moving up the loose sandy sloped terrain at $H_{opt} = 35$ cm. The maximum effective tractive effort $T_{D\text{max}}$ and the effective driving force $T_{rd}$ decrease remarkably with the increase of the slope angle $\beta$. The difference between $T_{rd}$ and $T_{D\text{max}}$ becomes large with the increment of $\beta$ due to the increasing slope resistances. In this case, it is clear
Fig. 9. (a) Relationship between effective tractive effort ($T_D$) and slip ratio ($i_{rd}$) for various slope angles ($\beta$). (b) Relationship between effective braking effort ($T_B$) and skid ($i_{rb}$) for various slope angles ($\beta$).
Fig. 10. (a) Relationship between total amount of sinkage \((s)\) of vehicle and slip ratio \((i_{rd})\) for various slope angles \((\beta)\). (b) Relationship between total amount of sinkage \((s)\) of vehicle and skid \((i_{rb})\) for various slope angles \((\beta)\).
Fig. 11. (a) Relationship among maximum effective tractive effort ($T_{D\text{max}}$), effective driving force ($T_{rd}$) and slope angle ($\beta$) for three kinds of eccentricity ($e$). (b) Relationship among effective braking effort ($T_{\text{B-20}}$), effective braking force ($T_{rb}$) and slope angle ($\beta$) for three kinds of eccentricity ($e$).
that the maximum slope angle of the terrain $\beta_{\text{max}}$ while pulling other construction machinery uphill is about 0.099 rad when the vehicle is operating at $e = 1/6$ and $H_{\text{opt}} = 35\ cm$, and that $T_{D_{\text{max}}}$ has a larger value for each slope angle when the amount of eccentricity of the center of gravity moves to the rear part.

On the other hand, Fig. 11(b) shows the relationship among the effective braking effort $T_{B_{-20}}$ at $i_{rb} = -20\%$, the corresponding effective braking force $T_{rb}$ and the slope angle $\beta$ of the terrain for the eccentricity $e = -1/6, 0$ and $1/6$ when the vehicle with rear-wheel brake is moving down the loose sandy sloped terrain at $H_{\text{opt}} = 0\ cm$. The absolute effective braking effort $|T_{B_{-20}}|$ decreases remarkably with the increment of the slope angle $|\beta|$ due to the slope resistance, whilst the effective braking force $|T_{rb}|$ decreases gradually with $|\beta|$. In this case, it is clear that $|T_{B_{-20}}|$ has a larger value for each slope angle when the amount of eccentricity of the center of gravity moves to the rear part of the vehicle. In the case of $\beta = -\pi/12\ rad$ and $e = -1/6$, $T_{B_{-10}}$ has a positive value which needs some traction force from other construction machinery.

Fig. 12 shows the relationship between the maximum effective tractive effort $T_{D_{\text{max}}}$ and the effective braking effort $T_{B_{-20}}$ at $i_{rb} = -20\%$ at the same values of $H = 35\ cm$ and the eccentricity of the center of gravity $e$, when the vehicle is moving up the terrain of slope angle $\beta = 0, \pi/72, \pi/36$ and $\pi/24\ rad$ during driving state of the rear wheel and is moving down the terrain of slope angle $\beta = 0, -\pi/72, -\pi/36$ and $-\pi/24\ rad$ during braking state of the rear wheel. During driving state of the rear wheel, the optimum eccentricity of the center of gravity $e_{\text{opt}}$ is determined as $1/6$ for the range of slope angle $0\leq \beta \leq \pi/24\ rad$. During braking action of the rear wheel, the optimum eccentricity of the center of gravity $e_{\text{opt}}$ is determined as $1/6$ for the range of slope angle $-\pi/24 \leq \beta \leq 0\ rad$.

5.2. Optimum application height

Fig. 13 (a) shows the relationship among the maximum effective tractive effort $T_{D_{\text{max}}}$, the corresponding effective driving force $T_{rd}$ and the slope angle $\beta$ of the terrain for the application height $H = 0, 15, 35\ cm$ when the two-axle, four wheel and rear-wheel drive vehicle is moving uphill on loose sandy sloped terrain at $e_{\text{opt}} = 1/6$. The maximum effective tractive effort $T_{D_{\text{max}}}$ and the effective driving force $T_{rd}$ decrease remarkably with the increment of the slope angle $\beta$. The difference between $T_{rd}$ and $T_{D_{\text{max}}}$ becomes larger with the increment of $\beta$ due to the increasing slope resistance. In this case, it is clear that the maximum slope angle of the terrain $\beta_{\text{max}}$ while pulling other construction machinery uphill is about 0.099 rad when the vehicle is operating at $e_{\text{opt}} = 1/6$ and $H_{\text{opt}} = 35\ cm$, and $T_{D_{\text{max}}}$ has a larger value for each slope angle when the position of the application height moves upward.

On the other hand, Fig. 13(b) shows the relationship among the effective braking effort $T_{B_{-20}}$ at $i_{rb} = -20\%$, the corresponding effective braking force $T_{rb}$ and the slope angle $\beta$ of the terrain for the application height $H = 0, 15, 35\ cm$ when the vehicle with rear-wheel drive is moving up the loose sandy sloped terrain at $e_{\text{opt}} = 1/6$. The effective braking effort $|T_{B_{-20}}|$ decreases remarkably with the increment of the slope angle $|\beta|$ due to the slope resistance, whilst the effective braking force $|T_{rb}|$
decreases gradually with $|\beta|$. In this case, it is clear that $|T_{B-20}|$ has larger values for each slope angle when the position of the application height moves downward.

Fig. 14 shows the relationship among maximum effective tractive effort ($T_{D_{\text{max}}}$), effective braking effort ($T_{B-20}$) and eccentricity ($e$) for various slope angles ($\beta$).

Fig. 12. Relationship among maximum effective tractive effort ($T_{D_{\text{max}}}$), effective braking effort ($T_{B-20}$) and eccentricity ($e$) for various slope angles ($\beta$).

During driving state of the rear wheel, the optimum application height $H_{\text{opt}}$ is determined as 35 cm for the range of slope angle $0 \leq \beta \leq \pi/24$ rad. During braking action of the rear wheel, the optimum application height $H_{\text{opt}}$ is determined as 0 cm for the range of slope angle $-\pi/24 \leq \beta \leq 0$ rad.
Fig. 13. (a) Relationship among maximum effective tractive effort ($T_{D_{\text{max}}}$), effective driving force ($T_{rd}$) and slope angle ($\beta$) for three kinds of application height ($H$). (b) Relationship among effective braking effort ($T_{b-20}$), effective braking force ($T_{rb}$) and slope angle $\beta$ for three kinds of application height ($H$).
6. Conclusions

The maximum effective tractive effort and the effective braking effort at skid -20% of a two-axle, four-wheel vehicle with rear-wheel drive and brake moving up and down a loose sandy sloped terrain was found to vary with the position of the center of gravity and the application height of the effective tractive and braking effort. The tractive performance of a 5.88 kN weight, rear-wheel drive and brake vehicle was simulated using terrain-wheel system constants. The optimum eccentricity of the center of gravity and the optimum application height of the effective tractive and braking effort should be determined for the terrain of various slope angles to obtain the largest value of the maximum effective tractive and braking effort.

Fig. 14. Relationship among maximum effective tractive effort ($T_{D_{max}}$), effective braking effort ($T_{B_{-20}}$) and application height ($H$) for various slope angles ($\beta$).
Several analytical results are summarized as follows:

1. During driving state of the rear wheel, the optimum eccentricity of the center of gravity $e_{\text{opt}}$ was found to be $1/6$ for the range of slope angle $0 \leq \beta \leq \pi/24$ rad. During braking action of the rear wheel, $e_{\text{opt}}$ was found to be $1/6$ for the range of slope angle $-\pi/24 \leq \beta \leq 0$ rad.

2. During driving state of the rear wheel, the optimum application height $H_{\text{opt}}$ was found to be $35$ cm for the range of slope angle $0 \leq \beta \leq \pi/24$ rad. During braking action of the rear wheel, $H_{\text{opt}}$ was found to be $0$ cm for the range of slope angle $-\pi/24 \leq \beta \leq 0$ rad.

3. The maximum slope angle of the terrain while pulling other construction machinery uphill was $0.099$ rad when the vehicle was operating at $e_{\text{opt}} = 1/6$ and $H_{\text{opt}} = 35$ cm, and the maximum effective tractive effort was a larger value for each slope angle when the amount of eccentricity of the center of gravity moved to the rear part of the vehicle and the position of the application height moved upward.

4. The absolute value of effective braking effort at skid $-20\%$ was larger for each slope angle when the amount of eccentricity of the center of gravity moved to the rear part of the vehicle and the position of the application height moved downward.

From these simulation results, an automatic control system for the optimum eccentricity of the center of gravity and the optimum application height can be expected by use of some inclinometer sensor mounted on the vehicle.

References


